Abstract:

Due to rising computing capacities, including and accounting for uncertain (model) parameters in numerical simulations is becoming more and more popular. Uncertainty Quantification (UQ) addresses this issue and provides a variety of different mathematical methods to quantify the influence of uncertain input parameters on numerical solutions and derived quantities of interest. This thesis is concerned with the development and improvement of different UQ methods for numerical simulations of compressible flow problems, described by random conservation laws like the compressible Euler or Navier-Stokes equations. We distinguish between polynomial-based (non-statistical) UQ methods and sampling-based (statistical) UQ methods.

The first part of this thesis investigates non-statistical UQ methods, in particular the Stochastic Galerkin (SG), Non-Intrusive Spectral Projection (NISP) and Stochastic Collocation (SC) method. While SG is a frequently used method for UQ of random partial differential equations, the classical SG approach is not ensured to preserve hyperbolicity of the underlying random hyperbolic conservation law. To this end we develop a hyperbolicity-preserving numerical scheme, which uses a slope limiter to retain admissible solutions of the SG system, while providing high-order approximations in physical and random space. The modified numerical scheme is applied to different challenging numerical examples for which the classical SG approach fails.

An important aspect when considering space-time-stochastic numerical schemes is to quantify the errors that arise from numerical discretization. In this thesis we derive a novel a posteriori error analysis framework for numerical discretizations of random hyperbolic systems of conservation laws, which rely on the Runge-Kutta Discontinuous Galerkin method in combination with polynomial-based UQ methods. Our estimates are based on the relative entropy framework of Dafermos and DiPerna and allow us to quantify the entire space-time-stochastic discretization error. Moreover, due to a splitting of the residual we are able distinguish between spatio-temporal and stochastic errors. Based on the a posteriori error estimates we design novel residual-based, space-stochastic adaptive numerical schemes. We confirm our theoretical findings by various numerical experiments.

The last part of this thesis is concerned with statistical UQ methods, especially Monte Carlo (MC) type methods. We extend the Multilevel Monte Carlo (MLMC) method to what we call hp-MLMC method. Instead of considering a hierarchy of spatially refined meshes, we allow for meshes which are arbitrarily hp-refined. The classical complexity analysis of MLMC is extended to the hp-MLMC method. Moreover, to increase the robustness and efficiency of an iterative version of hp-MLMC, we construct a confidence interval for the optimal number of samples per level. To demonstrate the efficiency of the hp-MLMC method combined with the novel sample estimator we apply our method to two different compressible flow problems described by the random Navier-Stokes equations. In particular, we consider an important problem from computational acoustics that exhibits physical phenomena with high sensitivity with respect to the problem parameters and which poses a challenging problem for UQ.