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Abstract

In this paper, we prove that each component of the Burnside ring of a finite group is the solvable component of the Burnside ring of a Weyl subgroup of its corresponding group, and we give some applications.

1 Preliminaries

Throughout this paper, G is a finite group. Its Burnside ring $B(G)$ is the Grothendieck ring of the category of finite left G-sets. This is the free abelian group on the isomorphism classes of transitive left G-sets of the form G/H for subgroups H of G, two such subsets being identified if their stabilizers H are conjugated in G. The addition and multiplication are given by the disjoint union and Cartesian product, respectively.

For a subgroup H of G we write $[H]$ for its conjugacy class. We write $V(G)$ for the set of conjugacy classes of subgroups of G and we give it the partial order in which $[H] \leq [K]$ if some conjugate of H lies in K.

In general, given a finite group G to which one associates in a natural way some other algebraic object (such as the Burnside ring $B(G)$, or the character ring, or the cohomology with coefficients in some fixed field, etc.), one asks whether or not one can recover the group from knowledge of the algebraic object associated to it. The general answer to such a question is no and the Burnside ring is no exception. For the particular case of the Burnside rings the specific question whether two finite groups having the same Burnside ring must be isomorphic was raised by Yoshida in [9] and the first counterexamples were found by Thévenaz in [8] (see also [1] for a counterexample involving p-groups). If one imposes further restrictions on the groups, then the answer can become affirmative, an instance of this occurring in [7] where it is shown that the answer to the above question is yes if we further know that both groups are abelian or Hamiltonian.

Knowing that, in general, the Burnside ring does not determine the group, one tries to find other invariants of the group which can be recovered from its Burnside ring. Very little is known in this direction. For example, it is not even known that if G is simple and G_1 is some other group such that $B(G) \cong B(G_1)$, then G is isomorphic with G_1. A finer invariant closely related to the Burnside ring is the so-called *table of marks*. This is the square matrix of order $n = \#V(G)$ whose entries are $\#[(G/H)^K]$, where $[H]$ and $[K]$ are elements of $V(G)$ and for a G-set X and a subgroup H of G we use X^H for the G-set of fixed points of X under the action of H. It is known that the above integer is well defined; i.e., does not depend on the particular representatives H and K for $[H]$ and $[K]$, respectively. The morphism

$$\nu_K : B(G) \to \prod_{[H]\in V(G)} \mathbb{Z}$$

given by $\nu_K(X) = |X^K|$ for all G-sets X is called *the mark corresponding to* K.

Kimmerle (see [2, Satz 7.5]) showed that the table of marks of G determines the composition factors for G. This result was rediscovered in [4]. In particular, if G is simple and G_1 is some other group having the same table of marks as G, then G_1 is isomorphic with G. As we have just said, this result is not known if one replaces the table of marks by the Burnside ring. It is known that the table of marks determines the Burnside ring but there is no known method to read a table of marks out of a Burnside ring.

In this paper, we prove a structure theorem for the Burnside ring. It is known that the Burnside ring is a product of *blocks*, each block being of the form $B(G)e_H$, where e_H is an idempotent associated to a perfect subgroup of G. Recall that H is perfect if H has no proper normal subgroups K such that the quotient group H/K is solvable. When $H = 1$ the block is called the *principal* block. Our main result shows (a little bit more than) that every block $B(G)e_H$ of $B(G)$ is isomorphic to the principal block $B(W_GH)e_1$ of the Burnside ring of the Weyl group N_GH/H of H. Such a result can be useful when dealing with isomorphisms of Burnside rings, which we illustrate by a couple of applications.

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2 Notation

Let G be a finite group and Π be a subset of all the prime numbers. We write $B_\Pi(G) = B(G)\otimes\mathbb{Z}_\Pi$ for the Burnside ring with coefficients in $\mathbb{Z}_\Pi$, where $\mathbb{Z}_\Pi$ is the ring of rational numbers a/b, with a positive integer b whose prime factors lie in Π. For a subgroup H of G we write $O^\Pi(H)$ for the smallest normal subgroup K of H such that H/K is solvable of order coprime to the primes in Π. We put $P_\Pi(G) = \{[H] \in V(G) \mid O^\Pi(H) = H\}$. For a subgroup H of G we let $e_{G,H}$ be the primitive idempotent in the *ghost ring*

$$\Omega(G) = \prod_{[H]\in V(G)} \mathbb{Z}$$

of G corresponding to H; i.e., of the form $e_{G,H} = (\delta_{HK})_{[K]}$, where $\delta_{HK} = 1$ if $[H] = [K]$ and 0 otherwise. Finally, for $H \in P_\Pi(G)$ we let $e^\Pi_{G,H}$ be the primitive idempotent of $B(G)$ corresponding to H, which by a result of Yoshida (see [10]) is given by

$$e^\Pi_{G,H} = \sum_{\substack{[K]\in V(G)\\ O^\Pi(K)=H}} e_{G,K}.$$

We also use the standard notation, namely that for $g \in G$ and $H \leq G$ we write ${}^gH$ for gHg^{-1} and N_GH for the normalizer of H in G. Finally, to simplify the notation, for a subgroup H of G we denote its Weyl group N_GH/H in G by W_GH.

3 Blocks of the Burnside ring

Our main result is the following.

Theorem 3.1. *We have*

$$B_\Pi(W_GH)e^\Pi_{W_GH,1} \cong B_\Pi(G)e^\Pi_{G,H} = \oplus_{\substack{[K]\leq V(G)\\ [O^\Pi(K)]=[H]}} \mathbb{Z}_\Pi G/Ke^\Pi_{G,H}, \tag{1}$$

In particular,

$$B_\Pi(G)e^\Pi_{G,1} = \oplus_{\substack{[K]\in V(G)\\ K\ \Pi-\mathtt{solvable}}} \mathbb{Z}_\Pi G/K \tag{2}$$

Proof. We first prove that that the sum in formula (1) is direct. Assume that an equation of the form

$$\sum_{[K]\in V(G)} \lambda_K G/Ke^\Pi_{G,H} = 0 \tag{3}$$

holds with some coefficients λ_K not all zero. Let $[L] \in V(G)$ be maximal with $[O^\Pi(L)] = [H]$ and $\lambda_L \neq 0$. We apply the mark ν_L to the above equation (3) and get $\lambda_L\nu_L(G/L) = 0$ because $\nu_L(e^\Pi_{G,H}) = 1$. Thus, $\lambda_L = 0$, which is a contradiction. Hence, the sum is direct.

We now only need to prove that the left hand side of (1) is contained in the right hand side of it. Notice that the relation

$$e^\Pi_{G,H} = \sum_{O^\Pi(K)\subseteq H} b_K G/K,$$

holds with some coefficients b_K, so for any subgroup $T \leq G$ we have

$$G/Te^\Pi_{G,H} = \sum_{O^\Pi(K)\subseteq H} c_K G/Ke^\Pi_{G,H}.$$

with some other coefficients c_K. Thus, it is enough to check that if the containment $[O^\Pi(K)] < [H]$ holds, then $G/Ke^\Pi_{G,H} = 0$. Assume therefore that $G/Ke^\Pi_{G,H} \neq 0$, take $L \leq G$ such that $\nu_L(G/Ke^\Pi_{G,H}) \neq 0$ and note that

$$0 \neq \nu_L(G/Ke^\Pi_{G,H}) = \nu_L(G/K)\nu_L(e^\Pi_{G,H}).$$

Hence, $[O^\Pi(L)] = [H]$ and $[L] \leq [K]$, therefore

$$[H] \leq [O^\Pi(L)] \leq [O^\Pi(K)] < [H].$$

This contradiction proves the stated equality.

Applying the equality (1) at $H = 1$ we get

$$B_\Pi(G)e^\Pi_{G,1} = \oplus \mathbb{Z}_\Pi G/Ke^\Pi_{G,1}.$$

By applying the mark ν_K with a solvable subgroup K to this last equality, we get $G/Ke^\Pi_{G,1} = G/K$.

We now consider the morphism

$$B_\Pi(W_GH)e^\Pi_{W_GH,1} = \oplus_{[O^\Pi(K)]=[H]}\mathbb{Z}_\Pi \frac{W_GH}{K/H} \xrightarrow{f} B_\Pi(G)e^\Pi_{G,H} = \oplus\mathbb{Z}_\Pi \frac{G}{K}e^\Pi_{G,H},$$

induced by

$$f\left(\frac{W_GH}{K/H}\right)=\frac{G}{K}e^{\Pi}_{G,H}$$

and extended linearly. It is easy to see that this morphism is well defined. On the other hand, it sends a basis into a basis since if $G/Ke^{\Pi}_{G,H}=G/Le^{\Pi}_{G,H}$ then $[K]=[L]$. Thus, $K=aLa^{-1}$ holds with some $a\in G$, therefore $H=O^{\Pi}(K)=aO^{\Pi}(L)a^{-1}=aHa^{-1}$, which implies that $a\in N_GH$, leading to the conclusion that

$$\frac{W_GH}{K/H}=\frac{W_GH}{L/H}.$$

The above argument shows that f is an isomorphism of abelian groups. We now check that this is a ring isomorphism as well. It clearly suffices to check it for the elements of a base. We have

$$f\left(\frac{W_GH}{K/H}\times\frac{W_GH}{L/H}\right)=\sum_{a\in\overline{K}\backslash W_GH/\overline{L}} f\left(\frac{W_GH}{K\cap^a L/H}\right), \tag{4}$$

where in the above formula (4) we used $\overline{K}$ and $\overline{L}$ for K/H and L/H, respectively, and the last sum above equals

$$\sum_{a\in K\backslash N_GH/L}\frac{G}{K\cap^a L}e^{\Pi}_{G,H}. \tag{5}$$

We now see that if

$$\frac{G}{K\cap^a L}e^{\Pi}_{G,H}\neq 0,$$

then there exists a subgroup $T\leq G$ such that $O^{\Pi}(T)=H$ and $T\leq K\cap^a L\leq^a L$. In this case, ${}^aH=H$, therefore $a\in N_GH$, and so

$$\sum_{a\in K\backslash N_GH/L}\frac{G}{K\cap^a L}e^{\Pi}_{G,H}=\sum_{a\in K\backslash G/L}\frac{G}{K\cap^a L}e^{\Pi}_{G,H}=\frac{G}{K}\times\frac{G}{L}e^{\Pi}_{G,H}. \tag{6}$$

Now (4)–(6) complete the proof of Theorem 3.1. □

We will denote by $B_S(G)$ the solvable component of the Burnside ring of G.

4 Normal subgroups and the Burnside ring

Throughout this section, we assume that G and G_* are two groups such that their Burnside rings are isomorphic. Let σ be an isomorphism of $B(G)$ onto $B(G_*)$. Building on work of Nicolson [5] and extending results of Kimmerle and Roggenkamp from [3] in which the group of automorphisms of the Burnside ring of a finite group was analyzed, in [6] it is shown that one may assume that this isomorphism is normalized; i.e., that $\sigma(G/1)=G_*/1$.

We give a generalization of these results for the components of the Burnside ring.

Given an isomorphism $\theta: B_S(G)\longrightarrow B_S(G')$ we can extend it to an isomorphism of the corresponding restricted ghost rings $\theta:\Omega_S(G)\longrightarrow\Omega_S(G')$, where

$$\Omega_S(G)=\prod_{\substack{[H]\in V(G)\\ H\ \texttt{solvable}}}\mathbb{Z}$$

Theorem 4.1. *Let G and G' be finite groups, and $\theta: B_S(G)\longrightarrow B_S(G')$ a normalized isomorphism. For any solvable subgroup D of G, let D' denote a subgroup of G' such that $\theta(e_{G,D})=e_{G',D'}$. Let V be a soluble subgroup of G. Then V' is soluble, $|V'|=|U|$, $|N_{G'}(V')|=|N_G(V)|$, and $\theta(G/V)=G'/V'+\sum_{T\in S_V}a_TG'/T$ where S_V is the family of soluble subgroups T of G' such that $|T|$ is a proper divisor of $|U|$.*

Proof. The proof is essentially the same as in [6] which is based in [3]. □

An isomorphism σ from $B(G)$ onto $B(G_*)$ induces a bijection σ_* of $V(G)$ onto $V(G_*)$. Note $\sigma_*([H]) = [H_*]$ provided the extension of σ to an isomorphism of the corresponding ghost rings maps $e_{G,H}$ to e_{G_*,H_*}. A natural question to ask is whether σ_* preserves the ordering. In general, the answer is no, and our first result here illustrates the obstruction to σ_* preserving the ordering. Throughout the following proposition and its proof p denotes a prime number.

Proposition 4.2. *Let $\sigma : B(G) \longrightarrow B(G_*)$ be an isomorphism of Burnside rings. Assume that σ is normalized and let σ_* denote the induced bijection from $V(G)$ to $V(G_*)$. Let $U \leq V \leq G$ be such that U is maximal in V of index p and $[\sigma_*(U)] \not\leq [\sigma_*(V)]$. Assume further that both U and V sit inside the block corresponding to the same perfect group P. Then the number $\#\{{}^gU \mid {}^gU \subset V\}$ is a multiple of p.*

Proof. By Theorem 3.1 we may assume that V is solvable; i.e., that $P = 1$ for if not we may replace G by W_GP. We now know that

$$\sigma(e_{G,V}) = e_{G_*,V_*}. \tag{7}$$

Applying σ to the well known formula

$$e_{G,V} = \frac{1}{|N_GV|}G/V - \beta(U,V)G/U + \sum_{\substack{[L]<[V] \\ [L]\neq[U]}} a_L G/L, \tag{8}$$

where $\beta(U,V) = \#\{{}^gU \mid {}^gU \subset V\}$ (see [10]) we arrive at

$$\sigma(e_{G,V}) = \frac{1}{|N_GV|}\sigma(G/V) - \beta(U,V)\sigma(G/U) + \sum_{\substack{[L]<[V] \\ [L]\neq[U]}} a_L \sigma(G/L). \tag{9}$$

Using (8) with V_* instead of V we get

$$e_{G_*,V_*} = \frac{1}{|N_{G_*}V_*|}G_*/V_* + \sum_{[L_*]<[V_*]} b_{L_*} G_*/L_*. \tag{10}$$

Using 4.1 and (7)–(10) together we arrive at

$$\sigma(G/V) = G_*/V_* + \frac{\beta(U,V)}{p}G_*/U_* + \sum_{\substack{|T_*|<|V_*| \\ [T_*]\neq[U_*]}} c_{T_*} G_*/T_*. \tag{11}$$

In this last equation we used the known fact that $|N_{G_*}V_*/V_*| = |N_GV/V|$. From (11) we immediately see that if $[U_*] \not\leq [V_*]$, then the coefficient of G_*/U_* must be an integer and therefore $p \mid \beta(U,V)$. □

An isomorphism σ from $B(G)$ onto $B(G_*)$ is called completely normalized if for each perfect subgroup P the subgroup P_* of G_* is again perfect. We record the following corollaries.

Corollary 4.3. *Suppose that $\sigma : B(G) \longrightarrow B(G_*)$ is a completely normalized isomorphism. Assume that V and N are subgroups of G sitting inside the block of the same perfect subgroup P; that is, $P \leq H \leq N_GP$ and H/P is solvable holds for $H \in \{V, N\}$. Assume further that N is normal in G and $N \leq V$. Then $N_* \leq V_*$.*

Proof. By Theorem 3.1, it follows that we may assume that V is solvable (otherwise, we may replace G by W_GP and both V and N by V/P and N/P, respectively). Let p be some prime such that V/N contains a normal subgroup of index p. Since V itself contains a normal subgroup of index p, it follows that the number of maximal subgroups of index p in V is not 1 (mod p) (see [3, Claim 2]). Hence, there exists a subgroup U of G such that $N \leq U \leq V$, the index of U in V is p and $\{{}^gU \mid {}^gU \subset V\}$ is not a multiple of p. Proposition 4.2 shows that $U_* \leq V_*$. Corollary 4.3 follows now by replacing V with U and by induction. □

Note that each isomorphism between Burnside rings maps the principal blocks in to each other. Thus, an immediate consequence of Corollary 4.3 is the following result.

Corollary 4.4. *$B(G)$ determines the lattice of the solvable normal subgroups of G.*

As final application, we describe the structure of the automorphism group of $B(G)$ for a general finite group G. Let $B_1, \ldots, B_m$ be representatives of the isomorphism types of the blocks of $B(G)$ with multipicity k_i. Clearly,

$$\mathrm{Aut(B(G))} = \prod_{\mathrm{i=1}}^{\mathrm{m}} \mathrm{Aut(B_i)} \rtimes \mathrm{S_{k_i}}.$$

Denote by P_i a perfect subgroup of G coresponding to B_i. By Theorem 3.1, we know that the B_i are the principal blocks of $N_i := N_G(P_i)/P_i$. Hence, we can apply the results on automorphisms of Burnside rings of finite solvable groups (see [3] and [6]). Using [5, Proof of Prop.3.4] and [3, 2.3 and 2.4], we get

$$\mathrm{Aut(B_i)} = \mathrm{Aut_n(B_i)} \rtimes \mathrm{X},$$

where $\mathrm{Aut_n(B_i)}$ denotes the group of normalized automorphisms of the block B_i and X depends on the normal subgroup structure of $N_G(P_i)/P_i$ as follows:

a) If $N_i \not\cong C_2 \times C_2 \times O$, where O is a group of odd order, then

$$X \cong C_2^{n_i},$$

with $n_i = a_i + b_i$, where

a_i is the number of odd primes p such that N_i has a unique subgroup of order p, and

$b_i = 1$ if N_i has a central subgroup of order 2 which is contained in each subgroup of order 4 of N_i, or which is the only subgroup of N_i of order 2; otherwise, $b_i = 0$.

b) If $N_i \cong C_2 \times C_2 \times O$, then

$$\mathrm{Aut(B_i)} = \mathrm{S_4 \times Aut\ O \text{ and } Aut\ O} = \mathrm{Aut_n(B_i)} \rtimes \mathrm{C_2^{a_i}}\ ,$$

where a_i denotes again the number of odd primes p such that N_i has a unique subgroup of order p.

Thus, the determination of Aut(B(G)) is reduced to the determination of normalized automorphisms of the principal block of the Burnside rings of its Weyl groups. These automorphisms induce on the other hand automorphisms of the subgroup lattice of the solvable normal subgroups of these Weyl groups, cf. Corollary 4.4. For computational aspects of the determination of such normalized automorphisms, we refer the reader to [3].

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