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1 Introduction

Let G be a finite group. Denote its integral group ring by $\mathbb{Z}G$ and let $V(\mathbb{Z}G)$ be the subgroup consisting of units with augmentation 1.
H. Zassenhaus stated with respect to torsion subgroups of units of $V(\mathbb{Z}G)$ three conjectures [28].

ZC–1 Let u be a unit of finite order of $V(\mathbb{Z}G)$. Then u is conjugate within $\mathbb{Q}G$ to an element of G.[1]

ZC–2 Let H be a subgroup of $V(\mathbb{Z}G)$ with the same order as G. Then H is conjugate within $\mathbb{Q}G$ to G.

ZC–3 Let H be a finite subgroup of $V(\mathbb{Z}G)$. Then H is conjugate within $\mathbb{Q}G$ to a subgroup of G.

These conjectures had a big influence on the development of the structure theory of integral group rings even if it turned out that ZC - 2 and ZC - 3 do not hold for all finite groups, for a recent survey we refer to [17]. The so-called first Zassenhaus conjecture ZC - 1 , originally stated in [34], is still wide open.

We say that the conjecture ZC - i (i = 1, 2 or 3) holds for a group G, if it is valid for $V(\mathbb{Z}G)$. The object of this note is a sketch of the proof of the following.

Proposition 1.1 *ZC - 1 is valid for all groups of order* ≤ 71.

The proof of the proposition uses most of the known results. In this sense this note presents also a survey on the first Zassenhaus conjecture. ZC - 1 might hold for all finite groups even if one has to say that the evidence (see the results mentioned in section 2) is small. Thus it makes sense to investigate with the known results and methods possible candidates for a counterexample. The first obvious question is to find a group of smallest order for which ZC - 1 is open. We remark in this context that in the mean time one knows that for ZC - 2 there exist counterexamples of order 96 and 144 [3], [10], [11, Kap.III], whereas the first discovered counterexample due to Roggenkamp and Scott was of order 14400 [26], [30].

2 Some known results

Probably the most powerful result on ZC - 1 known is the following due to A. Weiss.

Theorem 2.1 *[33] If G is nilpotent, then ZC–3 holds for G.*

Therefore in order to establish Proposition 1.1 we have not to consider groups of 16, 27, 32, 64.
ZC - 1 is open for metabelian even for metacyclic groups. Nevertheless in many special situations positive results for ZC - 1 have been established for such groups [23], [24]. These results are of course very useful for groups of small order. The most far theorem reaching with respect to metacyclic groups appears to be the following.

Theorem 2.2 *[31] Let $G \cong \langle a \rangle \rtimes X$, where $\langle a \rangle$ is a cyclic group of order n and X is an arbitrary abelian group of order m with $(m, n) = 1$. Then ZC–3 holds for G.*

The next three results also may be applied to several groups of small order.

Theorem 2.3 *[29] Let $G \cong A \rtimes X$, where A is an elementary abelian $p-$group and X is an arbitrary abelian group. If the action of X is faithful and irreducible on A, then ZC–1 holds for G.*

[1] Elements of G are called the trivial units of $\mathbb{Z}G$.

Theorem 2.4 *[22] Let $G \cong A \rtimes X$, where A is an arbitrary abelian $p-$group and X is of prime order q. If $|X| = q < p$ for all prime divisors p of $|A|$ and if q is also prime, then ZC–1 is true for G.*

Theorem 2.5 *[16] Let G be a Frobenius-group of order $p^m q^n$, with p and q prime. Then ZC–1 holds for G.*

We record two quick consequences of these theorems.

Proposition 2.6 *If the order of G is not divisible by a square of any prime, then ZC–3 holds for G.*

Also the next proposition is an immediate consequence of the above theorems.

Proposition 2.7 *If G is a group of order p^2q, where p and q are prime numbers, then ZC–1 holds for G.*

Recently M. Hertweck obtained the following result. It is not used in the proof of Proposition 1.1. It would however avoid some few calculations.

Theorem 2.8 *[12, Theorem 1.2] Let G be a finite group with normal Sylow p - subgroup S. Assume that S has an abelian complement. Then the first Zassenhaus conjecture holds for G.*

3 Direct Products

The ZC–1-question may be considered for arbitrary G - adapted [2] subrings R of $\mathbb{C}$.

(ZC-1)$_R$ Let R be an integral domain. We say that G satisfies $(ZC-1)_R$, if each torsion unit of RG with augmentation 1 is conjugated within $Q(R)G$ to an element $g \in G$, where $Q(R)$ denotes the quotient field of R.

Let C_i be a conjugacy class of the finite group G and $x = \sum_{g \in G} z(g)g \in \mathbb{Z}G$. Then the partial augmentation ν_i of x with respect to C_i is just

$$\nu_i = \sum_{g \in C_i} z(g).$$

If u is a torsion unit of $V(\mathbb{Z}G)$, then it is conjugate within $\mathbb{Q}G$ to a trivial unit if and only if for each power of u all partial augmentations except one vanish [22, Theorem 2.7], [19].
There is an analogon to this unique-trace property for torsion units in RG. Here, for an element $u \in RG$ the partial augmentations are defined in a natural way not within $\mathbb{Z}$, but within R.

Proposition 3.1 *Let $u \in V(RG)$ be a torsion unit of order k. Then u is conjugate within $Q(R)G$ to an element $g \in G$ if and only if for each divisor d of k there is a unique conjugacy class C_d of G with partial augmentation different from zero.*

The proof is similar to that of [19, Theorem 2] given by I. S. Luthar and I. B. S. Passi.

Proposition 3.2 *Let p be prim, ζ_p a primitive $p^{th}-$root of unity and G an arbitrary finite group. If $(ZC-1)_{\mathbb{Z}[\zeta_p]}$ is valid then ZC–1 holds for $G \times E$, where E is an elementary abelian $p-$group.*

[2] An integral domain R is called G - adapted, if no prime divisor of $|G|$ is invertible in R.

The main idea of the proof is to treat with different projection of $\mathbb{Z}(G \times C_p)$ onto $\mathbb{Z}[\zeta]G$ mapping a generator of C_p on the powers of a $p^{th}-$root of unity.

A similar result can be given if one replaces the prime p by $n \in \{4, 8\}$ and E by a multiple of C_4, respectively C_8.

We remark that it seems to be unknown whether ZC - 1 is valid for a direct product $G \times A$ with A abelian if it holds for G. From Proposition 3.2 we get the following special case.

Corollary 3.3 *If ZC–1 holds for a group G, then ZC–1 is true for $G \times E$, where E is an elementary abelian 2-group.*

As a summary of several known results we obtain now

Corollary 3.4 *If G is a finite subgroup of the orthogonal group $O(3, \mathbb{R})$, then ZC – 1 is valid for G.*

Proof. This is clear when G is abelian by Higman's thesis [13]. If G is a dihedral group then the result follows from [25], [20]. ZC - 1 for the tetrahedral group is proved in [1], for the octahedral group in [8] and for the icosahedral group in [19]. All other finite subgroups are a direct product of C_2 with one of the previous groups. Thus Corollary 3.3 completes the result.

Remark. Corollary 3.4 indicates that ZC − 1 might hold at least for groups with irreducible representations of small degree.

4 Groups of small order

By the results of section 2 the only possible group orders for a group G of order less or equal 72 for which ZC − 1 is still unknown are 24, 36, 40, 48, 54, 56, 60 and 72.

An examination – using GAP – of the groups of such orders by the results of section 2 and that ZC - 1 is valid for the symmetric group S_4 and the binary octahedral group [8], [7] rsp. yields a list of 30 groups up to order 60 which satisfy none of the theorems (and 18 groups of order 72). The remaining 30 groups up to order 60 for ZC–1 are now checked with ordinary characters by the Luthar - Passi method. For a more detailed explanation of this method we refer to [5] or [32].

There are some results that simplify the Luthar-Passi-Method. E.g. the following one due to Dokuchaev and Juriaans [7].

Proposition 4.1 *Let $N \trianglelefteq G$. Suppose ZC–3 holds for the factor G/N. Then any finite subgroup $H \leq V(\mathbb{Z}G)$ with $(|H|, |N|) = 1$ is conjugate to a subgroup of G.*

Within the Luthar - Passi method it is of course also used that the partial augmentations of non - trivial torsion elements of $V(\mathbb{Z}G)$ on central group elements have to be zero which is an immediate consequence of Berman's theorem [2], see also [27, Ch.5, p.102].
It turns out that the method of Luthar and Passi is a quite powerful tool for groups of small order. Together with Proposition 4.1 one gets enough information about the partial augmentations of the torsion units in $V(\mathbb{Z}G)$ to show the following result:

Proposition 4.2 *Let $|G| < 72$. Then ZC–1 holds for G, except G is isomorphic to one of the following groups:*

- $G_1 \cong GL(2, 3)$
- $G_2 \cong A_4 \rtimes C_4$

In case of G_1 torsion units of order 2, in case of G_2 torsion units of order 2 and 4 remain.

5 The two remaining groups

In this section we show that ZC–1 holds also for the groups G_1 and G_2. The arguments use information on the partial augmentations obtained by the Luthar - Passi method.

Let $u \in V(\mathbb{Z}G)$ be a torsion unit, $N \trianglelefteq G$ a normal subgroup and $\pi : G \longrightarrow G/N$ the reduction map. π also induces a map on the partial augmentations by adding up those partial augmentations whose corresponding conjugacy classes fuse in G/N.

Let $G \in \{G_1, G_2\}$ and $1 \neq N \trianglelefteq G$. Because of Proposition 4.2 $\pi(u)$ is conjugate to an element of G/N in $\mathbb{Q}(G/N)$ and therefore exactly one partial augmentation of $\pi(u)$ is different from zero.

1.) $G \cong GL(2,3)$

The head of the character table of G is given by (obtained by GAP):

i	1	2	3	4	5	6	7	8
$\lvert C_G(C_i)\rvert$	48	4	6	8	48	8	6	8
$o(r_i)$	1a	2a	3a	4a	2b	8a	6a	8b

Let $u \in V(\mathbb{Z}G)$ be a torsion unit of order 2. Let ν_i be its partial augmentation with respect to C_i. Then it is clear that

$$\nu_1 = \nu_3 = \nu_5 = \nu_7 = 0$$

The Luthar-Passi-method yields the following possible sets of partial augmentations that are in contradiction to ZC–1:

	ν_2	ν_4	ν_6	ν_8	χ_2	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
(a)	1	1	-1	0	(1,0)	(2,0)	(2,0)	(0,2)	(0,3)	(2,1)	(2,2)
(b)	-1	1	0	1	(1,0)	(2,0)	(2,0)	(0,2)	(2,1)	(0,3)	(2,2)
(c)	1	1	0	-1	(1,0)	(2,0)	(0,2)	(2,0)	(0,3)	(2,1)	(2,2)
(d)	-1	1	1	0	(1,0)	(2,0)	(0,2)	(2,0)	(2,1)	(0,3)	(2,2)
(e)	2	0	-1	0	(0,1)	(1,1)	(2,0)	(0,2)	(0,3)	(3,0)	(2,2)
(f)	2	0	0	-1	(0,1)	(1,1)	(0,2)	(2,0)	(0,3)	(3,0)	(2,2)
(g)	-1	0	1	1	(0,1)	(1,1)	(1,1)	(1,1)	(3,0)	(0,3)	(2,2)

where a tuple (a,b) on the right hand side of the table means $\mu_0(\chi_i,u) = a$ and $\mu_1(\chi_i,u) = b$ if $\mu_l(\chi_i,u)$ is the multiplicity of ζ^l as an eigenvalue of $D_i(u)$. Here ζ denotes a primitive 2^{nd}–root of unity and D_i is an ordinary representation affording χ_i.
The reduction map π modulo the center of G - the factor group is isomorphic to S_4, the symmetric group of degree four - gives the following two possible cases:

	χ_2	χ_3	χ_6	χ_7
$\pi(u) \sim (1,2) \in S_4$	(0,1)	(1,1)	(1,2)	(2,1)
$\pi(u) \sim (1,2)(3,4) \in S_4$	(1,0)	(2,0)	(1,2)	(1,2)

In each of the cases (a)-(g) we get a contradiction either to the eigenvalues of χ_6 or to those of χ_7.

Remarks. a) Recently M. Hertweck proved that if u is a torsion unit of $V(\mathbb{Z}G)$ - G denotes now again an arbitrary finite group - of order p^m then all partial augmentations of classes whose representatives are of order p^n with $n > m$ vanish. Together with [22, Theorem 2.7] this shows that if G has a unique conjugacy class of non-central elements of order p then ZC - 1 holds for all elements of order p. This gives another way for proving ZC - 1 for involutions of $V(\mathbb{Z}GL(2,3))$.
b) If $u = \sum_{g\in G} z(g)g$, then

$$\varepsilon_{(k)}(u) = \sum_{g\in G;\, o(g)=k} z(g)$$

is called the k - generalized trace of u. A. A. Bovdi conjectured [4], see also [15], that for a torsion unit $u \in v(\mathbb{Z}G)$ of order p^n

$$\varepsilon_{(p^n)}(u) = 1$$

and

$$\varepsilon_{(p^j)}(u) = 0 \text{ , if } j < n.$$

Note, if ZC - 1 holds for p - elements of a finite group G then Bovdi's conjecture follows. The result of Hertweck mentioned in a) shows that Bovdi's conjecture holds for elements of order p, therefore in particular for groups with elementary abelian Sylow subgroups. For further results on Bovdi's conjecture we refer to [15].

2.) $G \cong A_4 \rtimes C_4$

Using GAP the character table of G is

i	1	2	3	4	5	6	7	8	9	10
$\lvert C_G(C_i)\rvert$	48	8	48	6	16	8	8	6	16	8
$o(r_i)$	**1a**	**4a**	**2a**	**3a**	**2b**	**4b**	**4c**	**6a**	**2c**	**4d**
r_i^2	1a	2a	1a	3a	1a	2a	2c	3a	1a	2c
r_i^3	1a	4b	2a	1a	2b	4a	4d	2a	2c	4c
r_i^5	1a	4a	2a	3a	2b	4b	4c	6a	2c	4d
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	1	-1	-1	1	1	-1
χ_3	1	A	-1	1	1	-A	A	-1	-1	-A
χ_4	1	-A	-1	1	1	A	-A	-1	-1	A
χ_5	2	.	-2	-1	2	.	.	1	-2	.
χ_6	2	.	2	-1	2	.	.	-1	2	.
χ_7	3	1	3	.	-1	1	-1	.	-1	-1
χ_8	3	-1	3	.	-1	-1	1	.	-1	1
χ_9	3	A	-3	.	-1	-A	-A	.	1	A
χ_{10}	3	-A	-3	.	-1	A	A	.	1	-A

with $A = i$; r_i is a representative of the conjugacy class C_i

The same method as in 1.) may be applied for a torsion unit $u \in V(\mathbb{Z}G)$ of order 2, respectively of order 4. The involved normal subgroups in the case of $u^2 = 1$ are $N = Z(G) \cong C_2$ and $N \cong C_2 \times C_2$. This suffices to show that ZC–1 holds for involutions in $V(\mathbb{Z}G)$.
However in case of torsion elements of order 4 the reduction with respect to any normal subgroup does not lead to a final result. There are still remaining sets of partial augmentations, namely

	ν_2	ν_6	ν_7	ν_{10}	χ_9	χ_{10}
(a)	0	1	1	-1	(0,0,0,3)	(0,3,0,0)
(b)	1	0	-1	1	(0,3,0,0)	(0,0,0,3)

where the notation is similar to those in 1.).

To show that there is no torsion unit $u \in V(\mathbb{Z}G)$ with the properties (a) or (b) we work with a concrete faithful representation φ of the group G[3]. From now on u is as in (a) or (b).

Let

$$G \cong A_4 \rtimes C_4 \cong [(C_2 \times C_2) \rtimes C_3] \rtimes C_4 \;=\; [(\langle t_1\rangle \times \langle t_2\rangle) \rtimes \langle r\rangle] \rtimes \langle s\rangle.$$

The representation $\varphi : \; G \to \mathbb{C}^{3\times 3}$, defined via

[3]The idea for this solution was communicated to the first author by Martin Hertweck. We thank him for this hint.

$$\varphi(t_1) = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{pmatrix} \quad , \quad \varphi(t_2) = \begin{pmatrix} -1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & 1 \end{pmatrix}$$

$$\varphi(r) = \begin{pmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \end{pmatrix} \quad \text{and} \quad \varphi(s) = \begin{pmatrix} \cdot & i & \cdot \\ i & \cdot & \cdot \\ \cdot & \cdot & i \end{pmatrix}$$

is faithful and irreducible. Therefore either the corresponding representation of χ_9 or χ_{10} may be identified with φ. From the eigenvalues of u with respect to Wedderburn components of $\mathbb{C}G$ corresponding to these irreducible characters it follows that

$$U := \varphi(u) = \pm \begin{pmatrix} i & \cdot & \cdot \\ \cdot & i & \cdot \\ \cdot & \cdot & i \end{pmatrix}.$$

We show that this is impossible.

Clearly $U = \sum_{g \in G} u(g)\varphi(g)$ is a $\mathbb{Z}$ - linear combination of elements of $\varphi(G)$. Put $H = C_3 \rtimes C_4$. The coset decomposition of G with respect to the normal subgroup $C_2 \times C_2$ leads to $\varphi(G) = \bigcup_{h \in H} \varphi(C_2 \times C_2)\varphi(h)$. Because of $\varphi(s^2) = -E$ we get that U must be an element of

$$\left\langle \bigcup_{b \in B} D \cdot b \right\rangle_{\mathbb{Z}} = \bigcup_{b \in B} \langle D \rangle_{\mathbb{Z}}\, b,$$

where $D = \varphi(C_2 \times C_2)$ and $B = \{\varphi(id), \varphi(r), \varphi(r^2), \varphi(s), \varphi(r \cdot s), \varphi(r^2 \cdot s)\}$. Therefore

$$D = \left\{ \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{pmatrix}, \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{pmatrix}, \begin{pmatrix} -1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & 1 \end{pmatrix}, \begin{pmatrix} -1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -1 \end{pmatrix} \right\} \text{ and}$$

$$B = \left\{ \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{pmatrix}, \begin{pmatrix} \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & 1 \\ 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & i & \cdot \\ i & \cdot & \cdot \\ \cdot & \cdot & i \end{pmatrix}, \begin{pmatrix} i & \cdot & \cdot \\ \cdot & \cdot & i \\ \cdot & i & \cdot \end{pmatrix}, \begin{pmatrix} \cdot & \cdot & i \\ \cdot & i & \cdot \\ i & \cdot & \cdot \end{pmatrix} \right\}.$$

$\langle D \rangle_{\mathbb{Z}}$ consists only of diagonal matrices with integer entries. To get the entry $(U)_{1,1} = i$ the equation $U = d\varphi(r \cdot s) + q$ must be satisfied with $0 \neq d \in \langle D \rangle_{\mathbb{Z}}$ and $q \in \bigcup_{b \in B \setminus \{\varphi(r \cdot s)\}} \langle D \rangle_{\mathbb{Z}}\, b$. Because the matrix d is of the form

$$d = \begin{pmatrix} w+x-y-z & \cdot & \cdot \\ \cdot & w-x-y+z & \cdot \\ \cdot & \cdot & w-x+y-z \end{pmatrix} \text{ with } w, x, y, z \in \mathbb{Z},$$

we get the following equations:

$$\begin{aligned}
(U)_{1,1} = i \quad &= (d\varphi(r \cdot s))_{1,1} + (q)_{1,1} = (d)_{1,1}(\varphi(r \cdot s))_{1,1} + (q)_{1,1} \\
&= (w + x - y - z)i + (q)_{1,1} \\
(U)_{2,3} = 0 \quad &= (d\varphi(r \cdot s))_{2,3} + (q)_{2,3} = (d)_{2,2}(\varphi(r \cdot s))_{2,3} + (q)_{2,3} \\
&= (w - x - y + z)i + (q)_{2,3} \\
(U)_{3,2} = 0 \quad &= (d\varphi(r \cdot s))_{3,2} + (q)_{3,2} = (d)_{3,3}(\varphi(r \cdot s))_{3,2} + (q)_{3,2} \\
&= (w - x + y - z)i + (q)_{3,2}.
\end{aligned}$$

With $(q)_{2,3}, (q)_{3,2} \in \mathbb{Z}$ the last two equations imply

$$\begin{aligned} 2(w-x) = 0 \quad &\Rightarrow \quad w = x \\ 2(y-z) = 0 \quad &\Rightarrow \quad y = z \end{aligned}$$

and therefore $(d)_{1,1} = 2(w + y)$ and $(d)_{1,1}(\varphi(r \cdot s))_{1,1}$ is an even multiple of i. Because $(q)_{1,1} \in \mathbb{Z}$ this gives the desired contradiction. Therefore U can't be of the given form and a torsion unit u of $V(\mathbb{Z}G)$ with one of the properties (a) or (b) does not exist.

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