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Early explicit examples of non-desarguesian plane geometries

Markus Stroppel

Dedicated to the memory of Benno Artmann, who passed away on October 14, 2010.

The purpose of these notes is to collect information about the earliest examples of non-desarguesian planes, or, more generally plane geometries (where lines need not meet, and the parallel axiom need not be satisfied). Roughly since the third decade of the 20th century (definitely starting with Moufang's work [18], cf. [4]), non-desarguesian projective planes are considered as objects in their own right, and their systematic study has lead to an abundance of examples. For more information, consult Hall's seminal paper [6], the monographs [22], [13], [25].

The present notes concentrate mainly on the very first (known) examples. At the time of their construction, these examples were considered as pathological counterexamples showing the need for the embedding of the plane in a space of higher dimension or other additional structure in the axiomatic treatment of projective (or affine) planes.

The presentation will not be strictly chronological but starts with the first explicit treatment that actually occurred in print (as far as I know).

Hilbert 1899. A common opinion attributes to Hilbert the first explicit example of a non-desarguesian plane. In [9, § 23] Hilbert uses the ellipse passing through the points $E = (1, 0)$, $A = (0, 1/2)$, $B = (3/5, 2/5)$, $-E$, and $-A$. This ellipse also passes through $-B$ and $-E$. The construction of the new line system uses $F = (3/2, 0)$. See Fig. 1.

Hilbert writes ([9, pp. 54 f]): *"Da die Linienzüge, welche aus den genannten drei geraden Linien entspringen, sich, wie Figur 38 zeigt und wie man leicht durch Rechnung bestätigt, nicht in einem Punkte treffen, so folgt, dass der Desarguessche Satz in der neuen ebenen Geometrie für die beiden vorhin construirten Dreiecke gewiss nicht gilt."*

The figure in question consists of the circles k_1 through $F, A, -A$ and k_2 through $F, B, -B$. In order to see that said lines are not confluent we have to show that the second intersection point S of k_1 and the first axis (i.e., the one different from F) does not lie on k_2.

Since Hilbert explicitly gives the (rational!) coordinates of the points A, B, F it is indeed feasible to compute S, the midpoints M and N of the circles, and finally check that the radius of k_2 is different from the distance from N to S. The necessary computations can be done by hand (the use of a pencil and a not too small piece of paper is advisable, though).

Hilbert's example of 1899 has found recent interest, in particular, its automorphisms are studied in [1] and [26].

We note that reprints of the original "Festschrift" (i.e., the first edition [9] of Hilbert's "Grundlagen der Geometrie") are available, see [7, Ch. 5] or [12].

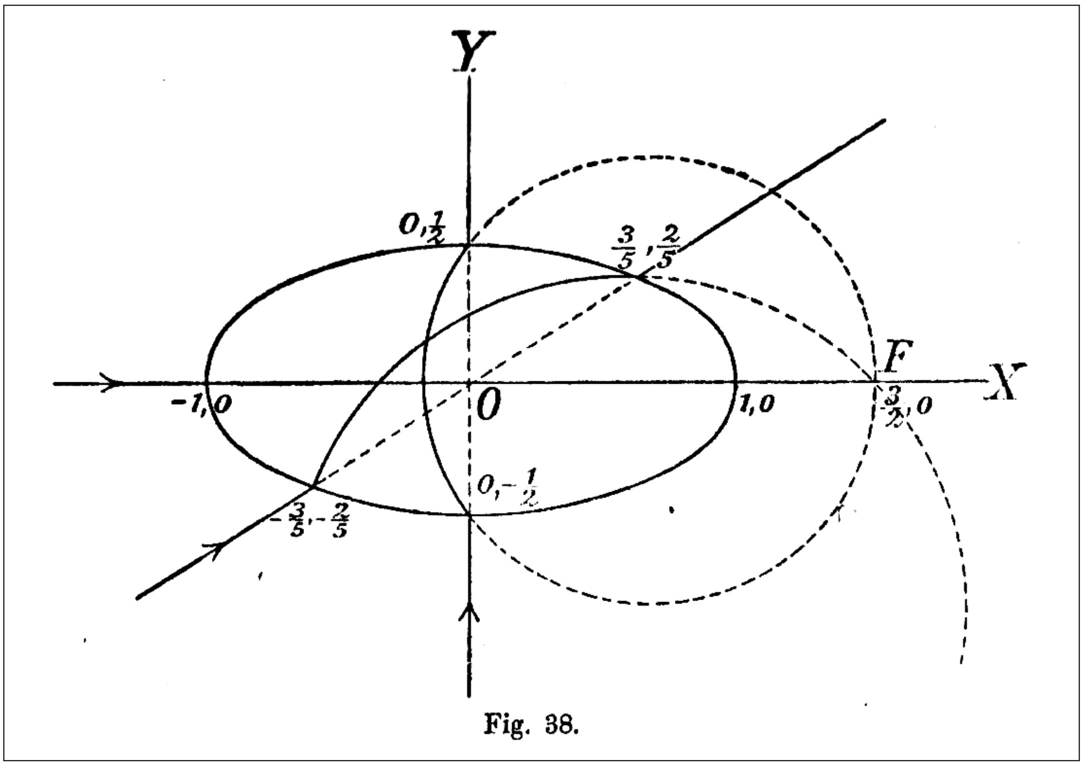

Figure 1: Hilbert's original drawing in [9, pp. 55].

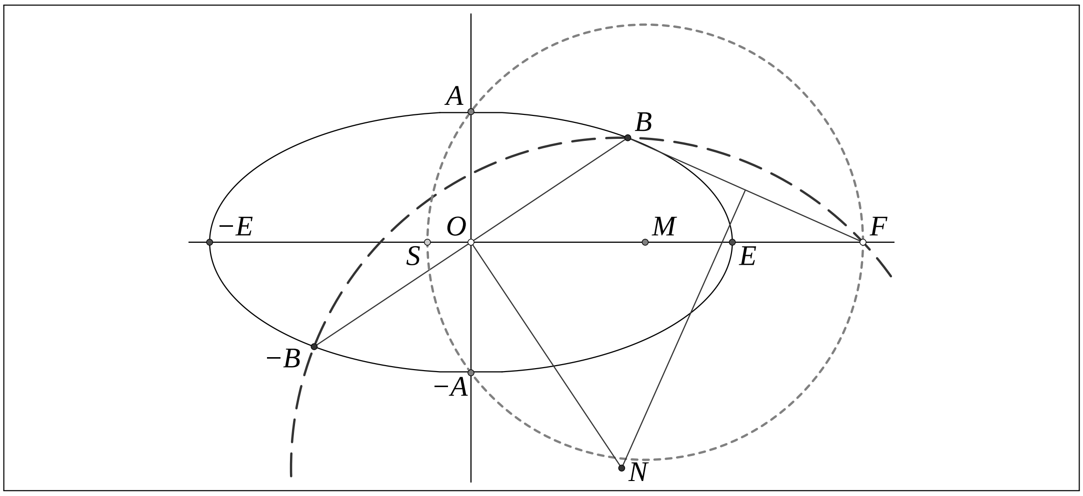

Figure 2: A proof that Hilbert's example is a non-closing Desargues configuration.

Hilbert 1898 a. The oldest *explicitly* described example of a non-desarguesian geometry (i.e., with an explicit proof that non-closing Desargues figures exist) known to me was given by Hilbert in a lecture course "Grundlagen der Euklidischen Geometrie" in 1898/99. Lecture notes elaborated by H. von Schaper ([8], see [7, Ch. 4] for a transcription) were mimeographed and circulated at the time, excerpts are also available through Toepell's publications ([30, p. 158 f], see also [31]). The example in question is discussed on pages 28–31 in [8], cf. [7, Ch. 4, p. 316], see Fig. 3.

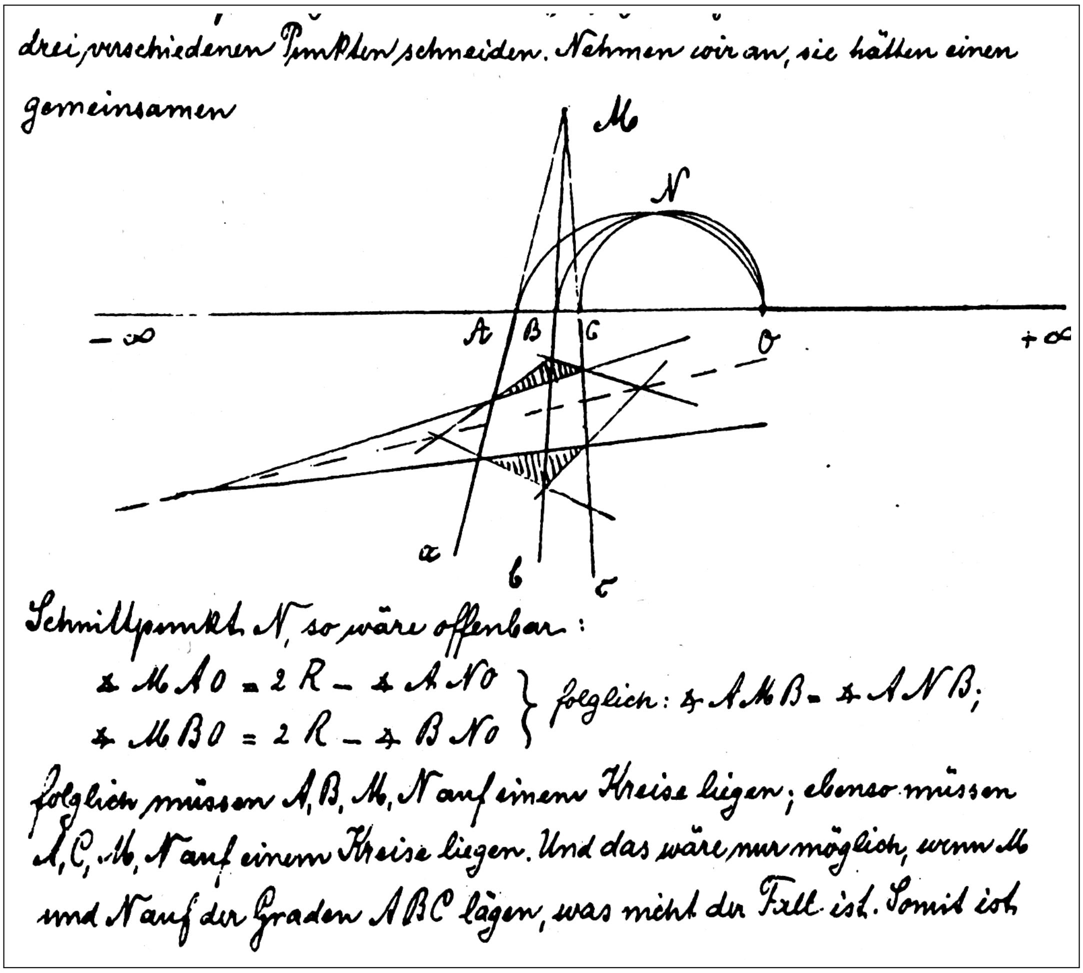

Figure 3: Hilbert's first example of 1898: [8, p. 30].

This early example is neither an affine nor a projective plane (and, unlike the hyperbolic plane, not an open subplane of any topological projective plane, cf. [29]). Hilbert apparently referred to the inscribed angle theorem (*"Peripheriewinkelsatz"*) in his proof that a certain Desargues configuration is not closed.

Hilbert 1898 b. In his lecture during the winter semester 1898/99, Hilbert has given a second example of a non-desarguesian plane ([8, p. 139], see Fig. 4 and [7, Ch. 4, p. 376]). Hilbert stresses the fact that this second example is an affine plane; it seems plausible that he did not have an affine example at hand when he started his lecture course.

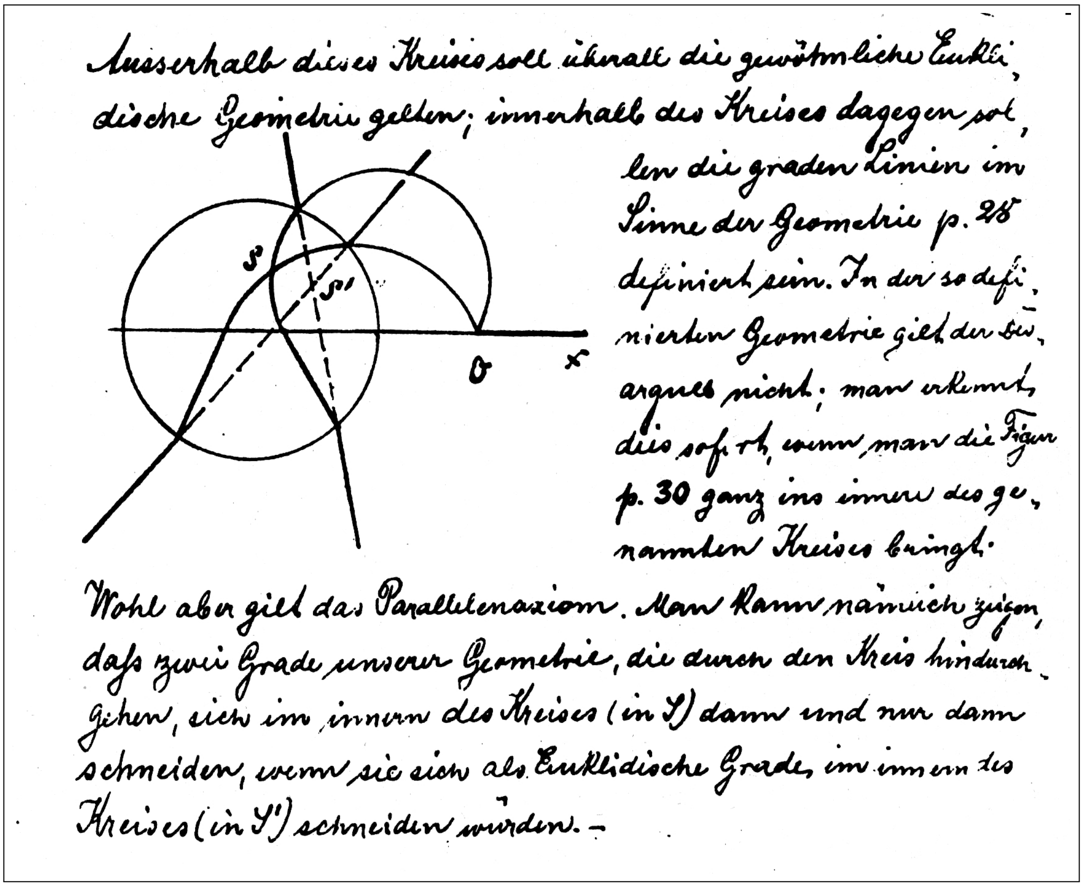
Ausserhalb dieses Kreises soll überall die gewöhnliche Eukli-
dische Geometrie gelten; innerhalb des Kreises dagegen sol-
len die graden Linien im
Sinne der Geometrie p. 28
definiert sein. In der so defi-
nierten Geometrie gilt der Des-
argues nicht; man erkennt
dies sofort, wenn man die Figur
p. 30 ganz ins innere des ge-
nannten Kreises bringt.
Wohl aber gilt das Parallelenaxiom. Man kann nämlich zeigen,
dass zwei Grade unserer Geometrie, die durch den Kreis hindurch-
gehen, sich im innern des Kreises (in S) dann und nur dann
schneiden, wenn sie sich als Euklidische Grade im innern des
Kreises (in S') schneiden würden. –

Figure 4: Hilbert's second example of 1898: [8, p. 139].

The plane that we find in the first editions of Hilbert's book [9] (see the comments to Hilbert 1899 above) may be interpreted as a further modification of the two constructions (1898 a, 1898 b).

Beltrami 1865. Apparently without mentioning Desargues' configuration, Beltrami [3] studied the question whether a (regular) surface allows a local diffeomorphism into the real affine plane such that geodesics are mapped into straight lines. He shows that (under suitable differentiability assumptions) this only happens if the surface has constant curvature. Thus Beltrami knew that there are plane geometries where the lines are shortest connections between their points but it is not possible to describe them with linear equations.

Klein 1873. Beltrami's result was known to Klein [14, Zweiter Abschnitt, § 2, p. 135 f] (cf. [15, p. 334], see Fig. 5). Like Beltrami, Klein did apparently not yet make the connection to Desargues' configuration. Clearly, he was well aware that one has to take a local point of view and consider a sufficiently small ("nicht zu ausgedehnt[. . .]") neighborhood in the surface.

die vorgetragene Behauptung kann geradezu dahin ausgesprochen werden: daß jedes den Voraussetzungen des Satzes entsprechende Flächensystem aus dem Systeme der Ebenen in dieser Weise erzeugt werden kann.

Was diesen Satz sehr merkwürdig macht, ist, daß ein analoger Satz, den man für die Ebene formulieren möchte, nicht existiert. Ist nämlich in einem begrenzten Teile der Ebene ein Kurvensystem von der Eigenschaft gegeben, daß durch je zwei Punkte eine und nur eine Kurve hindurchgeht, so bedarf es noch weiterer Bedingungen, ehe die Kurven durch lineare Gleichungen zwischen Punktkoordinaten dargestellt werden können. Diesen negativen Satz mag man aus einem Theoreme Beltramis ableiten. Ein Kurvensystem der gemeinten Art erhält man nämlich z. B., wenn man auf einer begrenzten [nicht zu ausgedehnten] einfach zusammenhängenden Fläche die geodätischen Kurven zieht und dann die Fläche auf einen Teil der Ebene beliebig ausbreitet. Aber Beltrami zeigt[24]), daß nur den Flächen von konstantem Krümmungsmaße die Eigenschaft zukommt, sich so auf die Ebene übertragen zu lassen, daß sich alle geodätischen Kurven mit geraden Linien decken. Man darf es daher auch nicht, wie seither wohl

Figure 5: Klein's reference [14, Zweiter Abschnitt, § 2, p. 135 f] to Beltrami

Wiener 1892. In a brief report [35] Wiener mentions that the axioms for projective planes do not allow to prove Desargues' (or Pascal's) Theorem: "Der Beweis dieser Sätze kann nicht aus den gegebenen Objecten und Operationen geführt werden". No further arguments are given, let alone an example constructed.

Peano 1894. Peano [20] (reprinted in [21, pp. 115–157]) alludes to a non-desarguesian plane geometry ([21, p. 139], see Fig. 6) obtained from the geodesics of a surface of non-constant curvature. Desargues' configuration appears under the name "triangoli omologici".

Apparently Peano did not further elaborate on his example; there seems to be no explicit description available. The authors of [2] make it plausible that Peano at least had the tools at hand to understand such an example. In any case, however, Peano's example would have been a "local" one, violating the parallel axiom (as in the hyperbolic plane).

Moulton 1902. Quite prominent is Moulton's example [19] of 1902. Hilbert uses this example from the seventh German edition [11] on, which appeared in 1930. However, Moulton's paper is already mentioned in a footnote to the first English translation [10, p. 74] which appeared in the same year as that paper, cf. [7, p. 417 f]. Moulton's example (together with natural generalizations) plays an important role in the recent theory of topological planes, see [25, Sect. 34].

Further examples. Another early example of a non-desarguesian plane is given in 1905 by Vahlen [33, p. 68 f]. In 1907 the first *finite* non-desarguesian planes that I know of have been published by Veblen and (Maclagan-)Wedderburn [34]. They use quasifields constructed by Dickson [5].

questo sarà un punto della retta *ef*; e così si ritrova il teorema di Desargues sui triangoli omologici, che si può enunciare:

Se fra i dieci punti $e, a, b, c, d, h, m, n, f, x$, di cui i primi quattro non sono complanari, passano nove fra le relazioni:
$h \varepsilon ad . h \varepsilon bc . e \varepsilon am . n \varepsilon ed . n \varepsilon mh . f \varepsilon mb . n \varepsilon cf . a \varepsilon xb . c \varepsilon xd . e \varepsilon xf$,
passerà pure la rimanente.

Di questi dieci punti uno comparisce nelle dieci relazioni, tre volte come interno; due punti compaiono ciascuno due volte come interni, ed una come estremi del segmento; tre punti compaiono ciascuno una volta come interni, e due come estremi; quattro punti compaiono sempre come estremi.

Dicesi che tre rette, non giacenti tutte in un stesso piano, appartengono ad una stessa stella, se a due a due sono complanari. Se due di esse si incontrano, allora anche la terza passa pel loro punto d'intersezione. La costruzione precedente permette di condurre la retta passante pel punto e, e appartenente alla stella determinata dalle due rette ab e cd.

Si dimostra il teorema dei triangoli omologici anche per punti di un piano; la costruzione precedente risolve il problema di unire il punto e col punto inaccessibile d'incontro di due rette date, e la figura è tutta contenuta nel foglio del disegno, se questo è rettangolare o almeno convesso.

Il teorema dei triangoli omologici, nel piano, è però conseguenza del postulato XV, e quindi è un teorema di Geometria solida. Che esso non sia conseguenza dei postulati precedenti, risulta da ciò che se per p intendiamo i punti di una superficie, e con $c \varepsilon ab$ intendiamo di dire che il punto c sta sull'arco di geodetica che unisce i punti a e b, allora sono verificati tutti i postulati dall'I al XIV, e non sussiste sempre la proposizione sui triangoli omologici. Questa proposizione continua però a valere per le superficie a curvatura costante.

Arrivati al teorema di Desargues, si può continuare senz'altro lo studio della Geometria di posizione, i cui principii sono così analizzati. Ora si possono introdurre, mediante definizioni, i punti impropríi o ideali, ecc. seguendo per es. il Pasch, nella sua opera citata.

Si è dimostrato il teorema V, il quale afferma che i piani che congiungono due rette giacenti in uno stesso piano con un punto fuori di questo piano, si incontrano secondo una retta. Per affermare che due piani qualunque, aventi un punto comune, hanno di comune una retta, occorre un nuovo postulato:

Figure 6: Peano's comments on Desargues' Theorem ([20], reproduced from [21, p. 139])

Cut and Paste. Hilbert's ideas of modifying the lines inside a bounded domain were discussed by Mohrmann [17] who distinguishes between the cases where the domain itself is non-desarguesian (possibly obtained as a suitable neighborhood in a surface of non-constant curvature) or where the geometry inside that domain is still desarguesian (and non-closing configurations only occur if points in- and outside that domain are involved). Tschetweruchin [32] has given a construction of an affine plane such that there is no desarguesian neighborhood; he replaces the lines of positive slope by curves of degree 3, cf. Fig. 7.

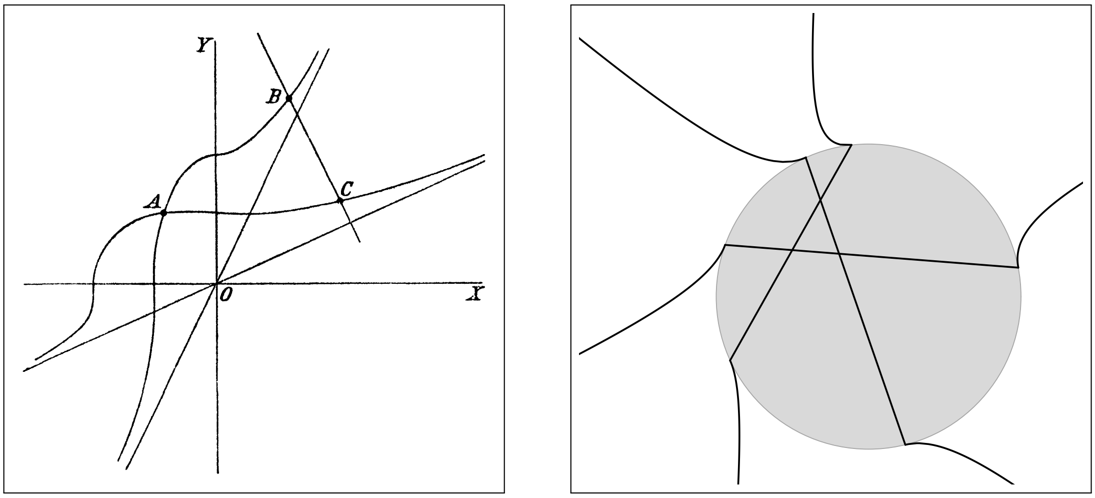

Figure 7: Tschetweruchin's [32] (left) and one of Salzmann's ([24], cf. [25, 35.1]) line systems.

Among the two-dimensional compact planes (i.e, projective planes with point spaces that are two-dimensional manifolds and one-dimensional submanifolds as lines) there is only one family of non-desarguesian planes such that the automorphism group contains a simple closed connected subgroup, see [24]. On each one of these planes, the group in question is isomorphic to the group $\Omega^+ \cong \mathrm{PSL}_2\,\mathbb{R}$ of proper hyperbolic motions, and acts in the usual way: an oval is left invariant. Inside the oval, the geometry is the usual (desarguesian) one, but outside the oval the lines are replaced by parts of hyperbolae. See Fig. 7. We note that this family of planes together with the family of Moulton planes comprises all two-dimensional compact planes with non-solvable automorphism group, see [25, 38.3].

More recently, the method of "cut and paste" has been systematically exploited for the construction of both projective planes and of circle geometries in [27], [28], [23] and [16]. Note that [16] is the first successful attempt to extend these methods to geometries whose lines are not one-dimensional (such as the projective plane over the complex numbers). Each one of these papers extends Hilbert's observation ([8, p. 139], cf. Fig. 4 or [7, p. 376 f]):

> "[...] *dass zwei Grade unserer Geometrie, die durch den Kreis hindurchgehen, sich im Innern des Kreises (in S) dann und nur dann schneiden, wenn sie sich als Euklidische Graden im Innern des Kreises (in S′) schneiden würden.*"

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