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Buttons, Holes and Loops of String: Lacing the Doily

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Abstract

Using graphical labels that encode the structure, we exhibit a combinatorial geometry for the group Sym(6) and use this to exhibit automorphisms of Sym(6) that are not inner. The labels are also used to facilitate the recognition of various graphs associated to the geometry.

1. The doily

The smallest generalized quadrangle has 15 points and 15 lines; every point or line is incident with three objects of the other type. An algebraic model is W(2), the *symplectic quadrangle* over the field $\mathbb{F}_2$ with 2 elements: the points and lines are the totally isotropic subspaces of a four-dimensional vector space over $\mathbb{F}_2$, with respect to the (essentially unique) non-degenerate alternating form on that space. We will not use this algebraic description in the sequel. In fact, we start from drawings and use labelings by drawings that encode combinatorial information. A reader interested in uniqueness of the smallest generalized quadrangle may for instance consult [8].

Usually, the smallest generalized quadrangle is pictorially represented by the *doily* (which seems to exist in two marginally different variants[1], see Fig. 1).

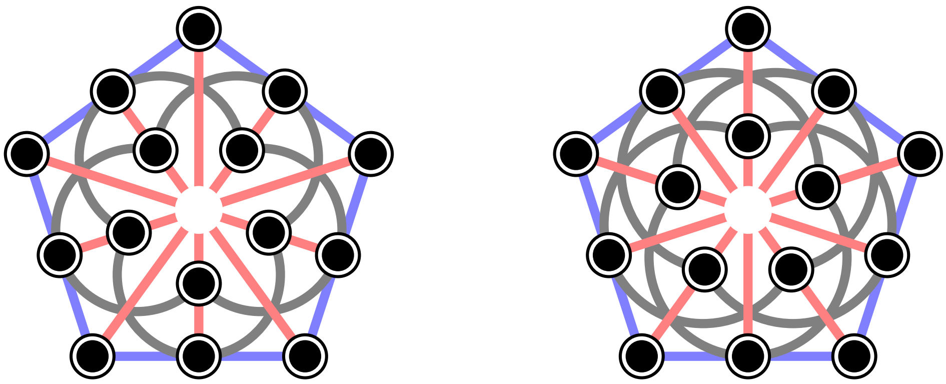


Figure 1: Two representations of the doily.

We will use the doily (i.e., the picture) and two different (but strongly interrelated) labelings of the points of the doily in the sequel to exhibit an action of the symmetric group Sym(6) on W(2),

[1] I do prefer the one shown on the left; ardent admirers of Belgian lace making may prefer the one on the right — as, apparently does some ancient artist in the desert of Oz [4].

determine the full group of automorphisms, and show that Sym(6) admits a strange (non-inner) automorphism.

2. Labeling by laces

From [3, 4.3, p. 54] (cf. also [4]) we take an idea to label the points in an ingenious way such that the full group of automorphisms becomes visible. The points are marked by the partitions of a six-element set (of "holes in a button") into three sets of size two (marked by loops of colored string through the holes), see the picture on the left in Fig. 2. Such a partition is called a partition of type $(2,2,2)$. Three such partitions label the points on a line if, and only if, they have one set of size two in common; we use that set as a label for the line.

The *drawing* in Fig. 2 allows the symmetries of a regular pentagon (viz., the group generated by a rotation of order five and a reflection). These symmetries also act on our labels; the label of the image of a point under one of these symmetries is obtained by application of the same symmetry to the label itself. In fact these labelings help to see *all* automorphisms of the doily; we use them in 2.1 to prove that Sym(6) acts on the doily by automorphisms and we will show later in 5.1 that there no more automorphisms.

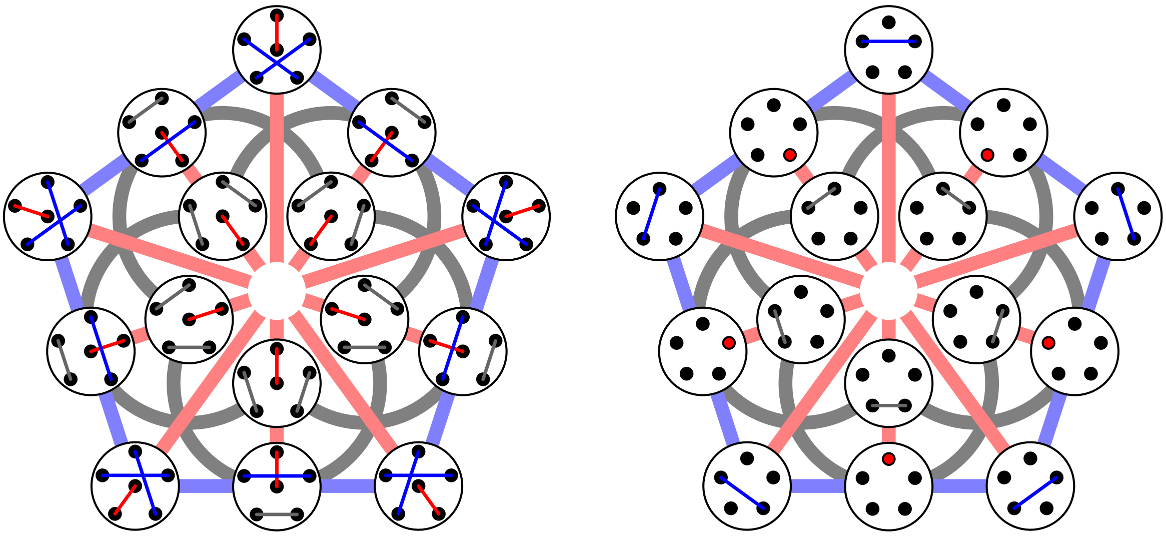


Figure 2: Labelings of the doily by buttons and laces

2.1 Theorem. *For every permutation $\sigma \in \mathrm{Sym}(6)$ we obtain permutations σ_L and σ_P of the set of subsets of size 2 and of the set of all partitions of type $(2,2,2)$, respectively, defined by $\{a,b\}^{\sigma_L} := \{a^\sigma, b^\sigma\}$ and $\{\{a,b\},\{c,d\},\{e,f\}\}^{\sigma_P} := \{\{a^\sigma,b^\sigma\},\{c^\sigma,d^\sigma\},\{e^\sigma,f^\sigma\}\}$.*

The pair (σ_P,σ_L) is an automorphism of the doily, i.e., a point labeled by the partition p and a line labeled by the set t are incident if, and only if, their images p^{σ_P} and t^{σ_L} are incident. Mapping σ to (σ_P,σ_L) is a group homomorphism from $\mathrm{Sym}(6)$ to the automorphism group of the doily. □

We will prove in 5.1 below that every automorphism of the incidence structure is induced by some element of Sym(6) in this action.

2.2 Labeling by involutions. We choose some set[2] Ω of order 6 identify each two-element subset $\{a, b\}$ of with the transposition $(a, b) \in \mathrm{Sym}(6)$, and every partition $\{\{a, b\}, \{c, d\}, \{e, f\}\}$ of type $(2,2,2)$ with the permutation $(a, b)(c, d)(e, f) \in \mathrm{Sym}(6)$; the permutations of the latter type are just the involutions without fixed points. Note that the action of Sym(6) on the partitions of type $(2,2,2)$ and on the subsets of size 2 may be interpreted as the action by conjugation on the set of involutions without fixed points and on the set of transpositions, respectively. Group theoretically, incidence between a point (labeled by a partition) and a line (labeled by a subset of size 2) means that the corresponding involutions commute with each other.

The picture on the right in Fig. 2 uses subsets of size one and two in a five-element set to mark the points. Each one of these $5 + 10 = 15$ subsets is used as a label. This new labeling uses precisely one color for each point (apart from black and white), we will talk of red, blue and gray points accordingly.

We think of the central hole as fixed (and not shown). Then the points are actually labeled by sets of two holes (the red ones by sets that contain the central hole). The lines correspond to partitions of type $(1,2,2)$ of the set of five non-central holes; we may equivalently think of them as partitions of type $(2,2,2)$ of the complete set of six holes. In this sense, the two labelings shown in Fig. 2 are dual to each other. The duality between (labelings of) points and lines that shows up here can in fact be expressed in terms of a polarity, see 3.1 below.

3. An example of a polarity

A *polarity* of an incidence structure is an involution interchanging points with lines in such a way that incidences are preserved. In general, an incidence structure will not admit any polarities at all. Even if polarities exist, they may be hard to visualize in pictures that show lines as sets of points; incidence graphs (see Section 6 below) are sometimes better suited for that purpose.

3.1 Proposition. *A polarity π of* W(2) *is given as follows (see the picture on the right in Fig. 2):*
- *Interchange each red point with the red line incident with it.*
- *Interchange each blue point with the blue line on the opposite side of the pentagon.*
- *Interchange each gray point with the gray line curving around it.*

3.2 Remark. The polarity can be given quite explicitly using a numbering of the holes in the buttons, see 5.3 below.

3.3 Remark. Some of the polarities of W(2) can also be seen as a symmetry of the incidence graph (see Fig. 5): for instance, the polarity π just described corresponds to the reflection at the vertical axis of symmetry there.

4. Ovoids and spreads

The set of red points in the picture on the right in Fig. 2 forms an "ovoid", i.e., a set of points such that every line contains precisely one of them (and, in particular, no two of them are collinear).

In the picture on the left in Fig. 2, the set of red lines (corresponding to the red laces which connect the central hole with any one of the others) forms a "spread", i.e. a set of lines such that every point is on precisely one of them.

[2] Later on, we use $\Omega = \{0, 1, 2, 3, 4, 5\}$.

Apart from the ovoid that consists of the red points, we find five other ones: each one of these corresponds to one of the non-central holes (marked green in Fig. 3). In fact, we obtain all the ovoids in this way:

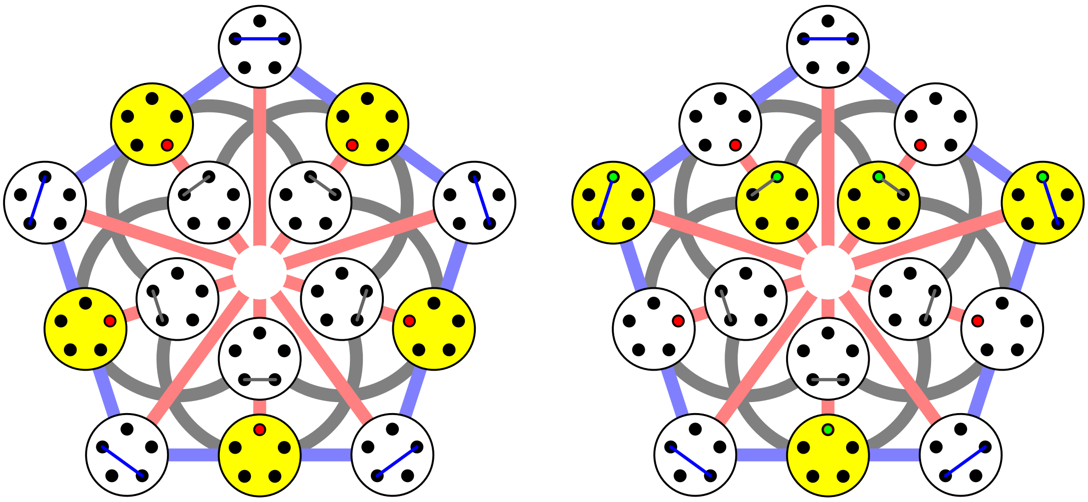


Figure 3: The ovoids in the doily: rotation of the one on the right yields four more.

4.1 Proposition. *There are precisely* 6 *ovoids in* W(2)*, namely those shown in Fig. 3.*

Proof. Using the labeling on the left in Fig. 2 we have constructed in 2.1 a transitive action of Sym(6) on the set of points. Thus it suffices to consider ovoids that contain a certain point p, we choose a red one. Further points of the ovoid have to be taken outside the three lines passing through p. If we also avoid the other red points we end up with an ovoid such as shown on the right in Fig. 3. So take a red point q as the second one. Then three more points (collinear with q) are excluded, and there remain four points to choose from: three of them are red, one is gray. The gray one is collinear with each of the three red ones. Thus we have to avoid it, and end up with the ovoid shown on the left in Fig. 3. □

4.2 Corollary. *There are precisely six spreads in* W(2)*; namely, the images of the ovoids under a polarity (such as* π *from 3.1).* □

We leave it as an exercise to the reader to draw pictures to show the spreads, see Fig. 4.

Figure 4: Wondering about spreads in the doily.

5. The full group of automorphisms

5.1 Theorem. *The group* Sym(6) *in its action on the partitions of type* $(2,2,2)$ *is the full group of automorphisms of* W(2)*.*

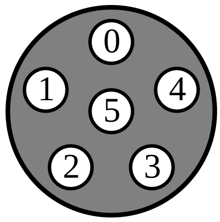
Proof. The full group Γ of automorphisms acts on the set of 6 ovoids, see 4.1. If we fix all these ovoids, we fix each point. Thus the action on the ovoids gives an injective homomorphism from Γ into Sym(6), and $|\Gamma| \le 6!$ follows. Conversely, we have constructed in 2.1 an injective homomorphism from Sym(6) into Γ, mapping σ to $(\sigma_P, \sigma_L) \in \mathrm{Sym}(6)$. Thus $6! \le |\Gamma|$, and the homomorphism in 2.1 is an isomorphism. □

A different approach (using various graphs derived from the doily) is described in [5].

5.2 Theorem. *The group* Sym(6) *has an automorphism that is not inner.*

Proof. In fact, conjugation by the polarity π from 3.1 induces an automorphism $\tilde{\pi} \in \mathrm{Aut}(\Gamma)$ of the full group $\Gamma = \mathrm{Sym}(6)$ of automorphisms of W(2). This automorphism of Sym(6) interchanges involutions (labeling points of the doily) that have no fixed points with transpositions (labeling lines). Thus it is not inner. □

5.3 Examples. We exhibit the action of $\tilde{\pi}$ (i.e., conjugation by π) on some more elements of Sym(6), using the identification from 2.2. Thus we refer to the involutions that fix no hole as "points" and to transpositions as "lines". For the sake of easy reference, we number the six holes, as shown in the drawing on the right. Using this numbering, one can also give an explicit description of the polarity π:



- Each red line is of the form $(5, j)$ for some $j \in \{0,1,2,3,4\}$. The image under π is $(5, j)(j+1, j-1)(j+2, j-2)$ where addition in $\{0,1,2,3,4\}$ is performed modulo 5.
- Each blue line is of the form $(j, j+2)$ for $j \in \{0,1,2,3,4\}$; the image under π is $(5, j+1)(j, j-2)(j+2, j-1)$.
- Each gray line is of the form $(j, j+1)$ for $j \in \{0,1,2,3,4\}$; the image under π is $(5, j-2)(j, j-1)(j+1, j+2)$.

Note that a point and a line are incident if, and only if, they commute in the group Sym(6). Moreover, two points (or lines) commute if there exists a point (or a line, respectively) that is collinear (confluent) with both. These relations can more conveniently be formulated in terms of the incidence graph, see 7.1 below.

We are ready for a case study of all conjugacy classes in Sym(6):

(1) We already know that $\tilde{\pi}$ interchanges lines (transpositions) with points.

(2) Applying $\tilde{\pi}$ to the product of an incident point-line pair[3] we see that $\tilde{\pi}$ leaves the set of double transpositions invariant. A member α of this class is centralized (and thus inverted) by π precisely if the unique flag fixed by α is an absolute one. This happens precisely if $\alpha \in \{(01)(24), (12)(30), (23)(41), (34)(02), (40)(13)\}$.

(3) A three-cycle can be written as the product of two non-commuting transpositions. These two transpositions are lines that have no point in common; the polarity π maps them to points that are not collinear with a common point. In other words, the image of the three-cycle is an element of order three that fixes no point.

[3] In fact, the double transpositions may be used as labels for the edges of the incidence graph, see 7.1 below.

To be more explicit, consider the (red) lines $(5,0)$ and $(5,1)$. Their images under π are the (red) points $(5,0)(1,4)(2,3)$ and $(5,1)(0,2)(3,4)$, respectively. Thus $\tilde{\pi}$ maps[4] $(5,0,1) = (5,0)(5,1)$ to $(5,0)(1,4)(2,3)\,(5,1)(0,2)(3,4) = (5,2,4)(0,1,3)$.

(4) Every four-cycle belongs to $\mathrm{Sym}(6)\setminus\mathrm{Alt}(6)$ while the other elements of order 4 in $\mathrm{Sym}(6)$ lie in the characteristic subgroup $\mathrm{Alt}(6)$. Thus the image of any four-cycle under $\tilde{\pi}$ is a four-cycle, again. Assume that π normalizes the subgroup $\langle\psi\rangle$ generated by an element ψ of order 4. If ψ is a four-cycle then π centralizes $\langle\psi\rangle$. If ψ is a conjugate of $(0123)(45)$ then π induces inversion on $\langle\psi\rangle$. Note also that $\langle\psi\rangle$ is normalized by π if, and only if, the square $\alpha := \psi^2$ is centralized by π, cf. case (2). Thus the four-cycles centralized by π are the conjugates of (0214) and (0412) under $\langle(01234)\rangle$ while the conjugates of $(0214)(35)$ and $(0412)(35)$ under $\langle(01234)\rangle$ are inverted by π.

(5) Every five-cycle φ fixes precisely one ovoid $\mathscr{O}_\varphi$ and precisely one spread $\mathscr{S}_\varphi$. The spread is easy to find: it consists of those subsets of size two that involve the hole fixed by φ. Now π normalizes $\langle\varphi\rangle$ if, and only if, it interchanges $\mathscr{O}_\varphi$ with $\mathscr{S}_\varphi$.
The five-cycle $(0,1,2,3,4)$ has the spread $\mathscr{S}_{(0,1,2,3,4)} = \{(5,0),(5,1),(5,2),(5,3),(5,4)\}$, the ovoid $\mathscr{O}_{(0,1,2,3,4)}$ is the image of $\mathscr{S}_{(0,1,2,3,4)}$. Using $(0,1,2,3,4) = (0,1)(0,2)(0,3)(0,4)$ one computes the image under $\tilde{\pi}$; it turns out that π centralizes $(0,1,2,3,4)$. One can infer this also directly from the definition of π in 3.1. Every five-cycle with the same spread but not in $\langle(0,1,2,3,4)\rangle$ generates a cyclic group that is not normalized by π.
Now consider a five-cycle fixing a hole $j < 5$. Since $\langle(0,1,2,3,4)\rangle$ centralizes π, it suffices to consider the case $j = 0$. We find that $\tilde{\pi}$ maps $(1,2,3,4,5) = (1,2)(1,3)(1,4)(1,5)$ to its inverse, namely $(1,5,4,3,2) = (0,1)(2,3)(4,5)\,(5,2)(0,3)(1,4)\,(5,0)(1,3)(2,4)\,(5,1)(0,2)(3,4)$. This means $\mathscr{O}^{\pi}_{(1,2,3,4,5)} = \mathscr{S}_{(1,2,3,4,5)}$. Again, we note that every five-cycle with the same spread but not in $\langle(1,2,3,4,5)\rangle$ generates a cyclic group that is not normalized by π.

(6) It remains to study the two classes of elements of order 6, represented by $(0,1,2,3,4,5) = (0,1)(0,2)(0,3)(0,4)(0,5)$ and $(0,1,2)(4,5)$, respectively. One obtains that $\tilde{\pi}$ interchanges the two classes; in fact, we have $\pi(0,1,2,3,4,5)\pi = (0,4,5)(1,3)$. We could also use our observation (3): the members of the different classes of elements of order 6 have squares in different classes of elements of order 3, and the latter are interchanged by $\tilde{\pi}$.

The stabilizer $\Sigma \cong \mathrm{Sym}(5)$ of hole 5 acts transitively on the set of points in the picture on the left in Fig. 2, and it acts transitively on the set of lines in the picture on the right. This shows:

5.4 Proposition. *There are (at least) two different conjugacy classes of groups isomorphic to* $\mathrm{Sym}(5)$ *in* $\mathrm{Sym}(6)$*; in a given action of* $\mathrm{Sym}(6)$ *on the doily each member of one of these classes preserves an ovoid (and is transitive on the set of lines) and each member of the other class preserves a spread (and is transitive on the set of points).* □

5.5 Dualities. If δ is a duality of some incidence structure then every other duality is obtained as a product $\delta\alpha$ with an automorphism α. If δ is a polarity then the product $\delta\alpha$ is a polarity if, and only if, the conjugate $\delta^{-1}\alpha\delta = \delta\alpha\delta$ is the inverse of α.

The doily is just the start of the infinite family of (finite) *symplectic quadrangles* $\mathrm{W}(q)$ where q is a prime power. Note that $\mathrm{W}(q)$ admits dualities precisely if q is even, and that $\mathrm{W}(q)$ admits polarities precisely if q is even and not a square, cf. [6, 4.9]. One knows that there is just one conjugacy class of polarities, cf. [6, 5.4] (and use the fact that there is at most one Tits endomorphism in a *finite* field).

[4] The product of cycles should be read from left to right, like the rest of this text.

From the case study in 5.3 one could also derive directly that there is only one conjugacy class of polarities of the doily.

5.6 Remark. Among all symmetric groups, the group Sym(6) is singled out by the existence of automorphisms that are not inner. For a nice proof, see [1]. An analogous result holds for the alternating groups, see [2], cf. [10, 2.4.1]. See [10, 2.4.2] for a purely group theoretic construction of an outer automorphism of Sym(6) (using the fact that Sym(5) has 6 Sylow 5-subgroups).

6. The incidence graph

For any incidence structure $\mathscr{G}$ (with point set P and line set L) the *incidence graph* is the bi-partite graph with vertex set the disjoint union of the point and line sets, and an edge between two vertices precisely if the two are incident in $\mathscr{G}$.

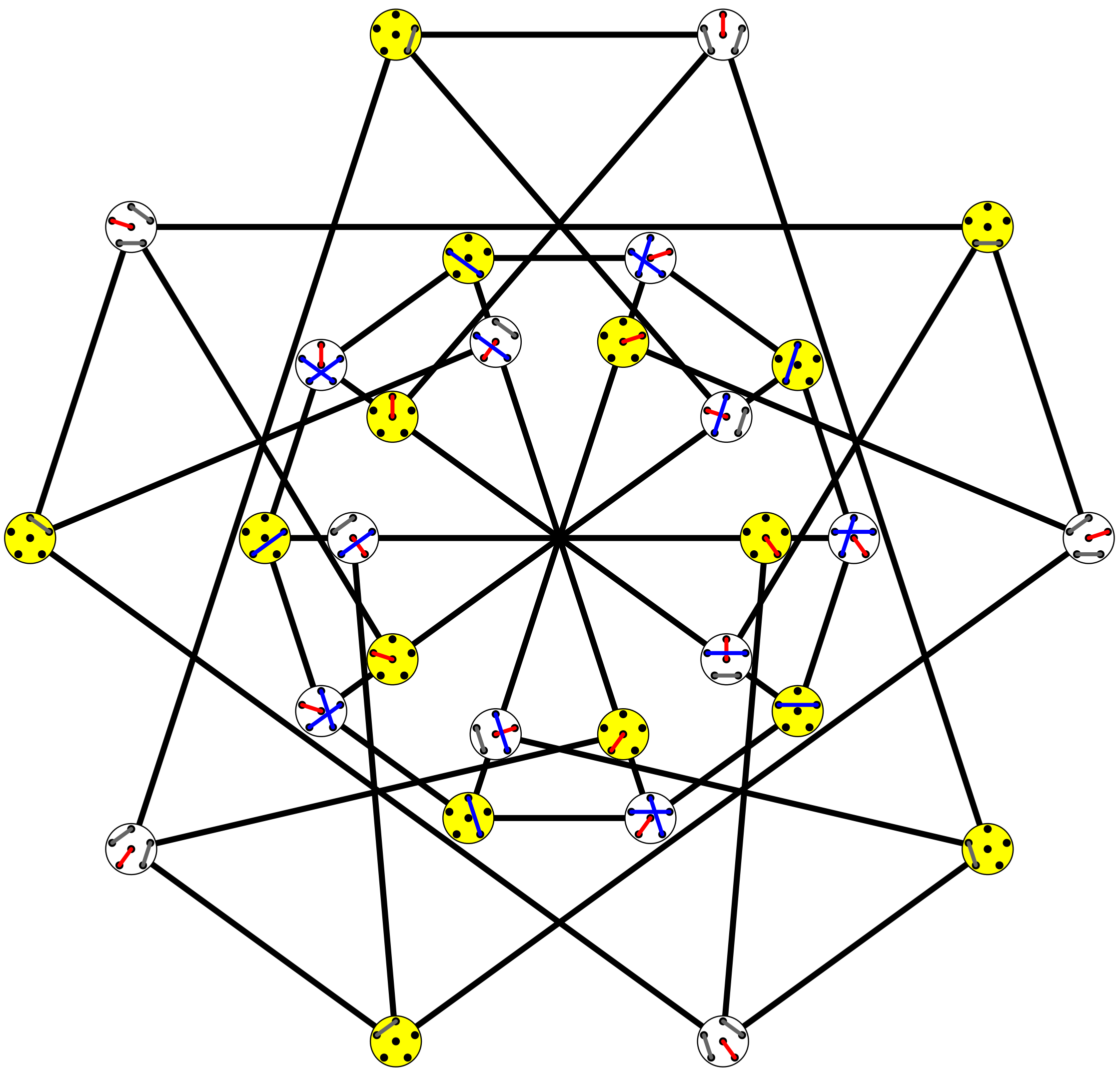


Figure 5: The incidence graph of W(2), with rotationally symmetric labels.

The graph shown in Fig. 5 and in Fig. 6 is isomorphic to the incidence graph of W(2); the labels suggest two isomorphisms explicitly. The labeling in Fig. 5 actually shares the rotational symmetry with the picture in Fig. 2. Some of the polarities of W(2) can be seen as symmetries of this pictorial representation of the incidence graph; for instance, the polarity π from 3.1 corresponds to the reflection at the vertical axis of symmetry in Fig. 6.

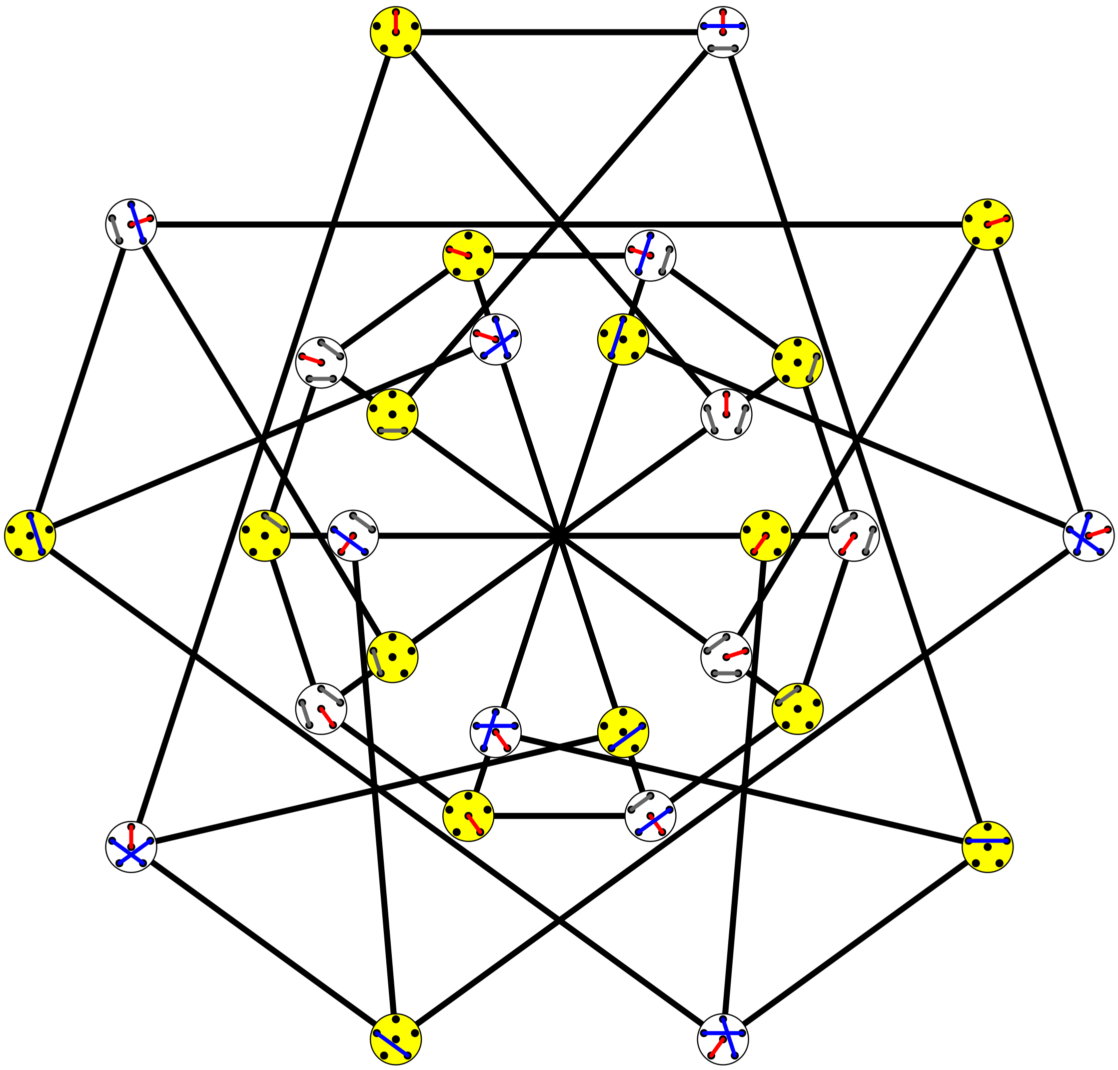


Figure 6: The incidence graph of W(2), the polarity π is the reflection at the vertical axis.

Fig. 7 gives another representation of the incidence graph (due to Jacques Tits [9]). The labels of the vertices are adapted to our present conventions; they should be read as two-element subsets of $\{0,1,2,3,4,5\}$ — this is the labeling of lines in 2.1, cf. 5.3. Each label is used twice; the polarity interchanges opposite vertices in this picture of the graph (i.e., those labeled with the same subset). Fig. 8 shows the same picture with our labels from Fig. 2.

See [4, Fig. 12] for yet another representation of this incidence graph.

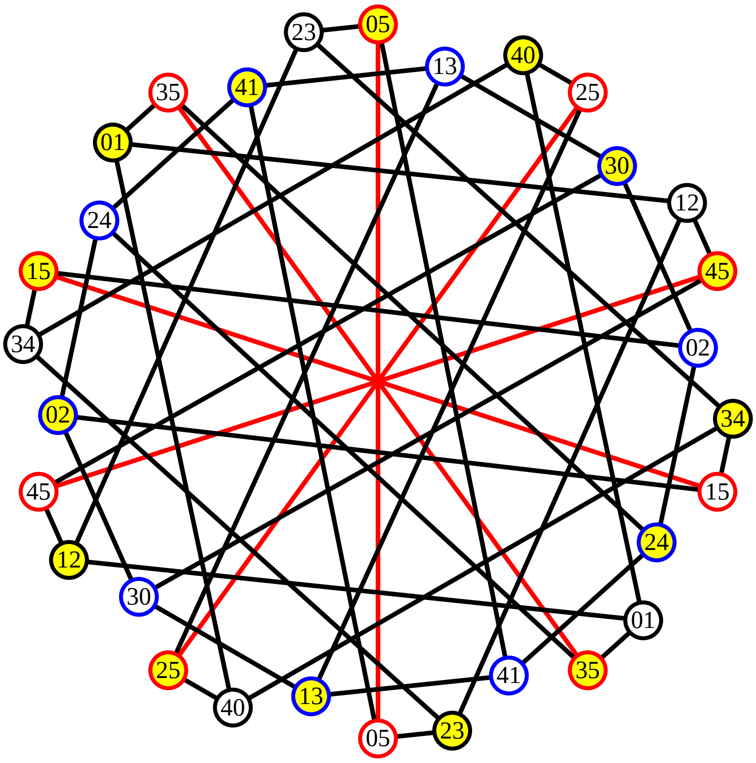


Figure 7: The incidence graph of W(2): Jacques Tits' version [9].

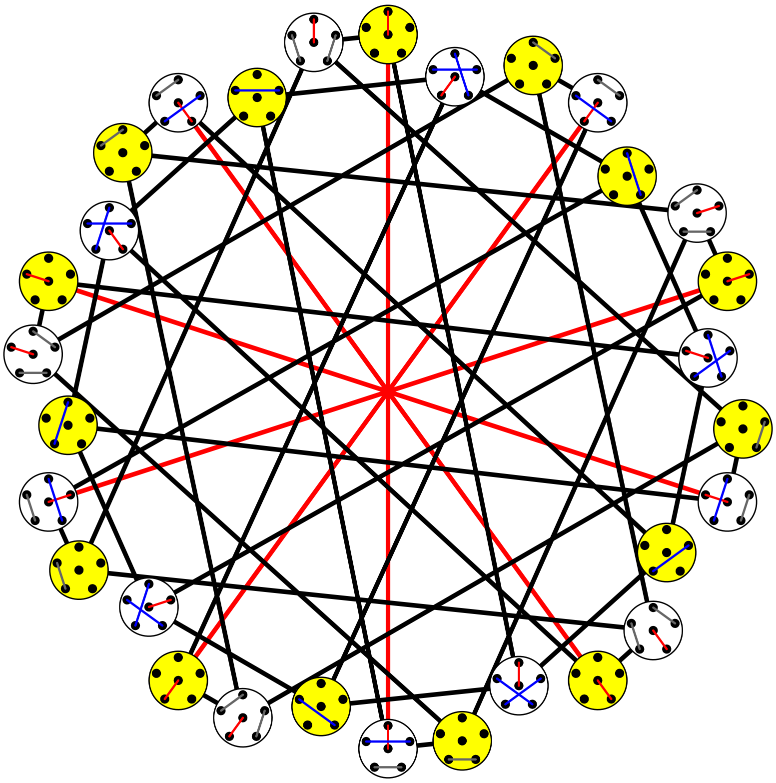


Figure 8: The incidence graph of W(2): Jacques Tits' version, with our labels.

6.1 Remarks. Two vertices in the incidence graph of the doily are adjacent if, and only if, the corresponding involutions belong to different conjugacy classes and their product is an involution (necessarily a double transposition). Thus the edges of the incidence graph may be labeled by the double transpositions; the edge $(a,b)(c,d)$ then joins $(a,b)(c,d)(e,f)$ and (e,f) for $\{e,f\} := \{0,1,2,3,4,5\} \setminus \{a,b,c,d\}$.

Two transpositions in Sym(6) commute if, and only if, the corresponding vertices in the incidence graph are joined by a path of length 2.

7. The line graph

We associate yet another graph with the doily, see Fig. 9: the vertices correspond to the lines in the doily, and an edge joins two vertices if the corresponding lines are confluent (i.e., meet in a point). This graph is known as the *line graph* (or *confluence graph*).

The location of the vertices in Fig. 9 differs from that in (the dual of) the doily in order to make all edges of the graph easily visible. The colors are those that have been used for the lines (and, via the polarity π, also for the points) in Fig. 2. Moreover, each edge has the color of the intersection point that caused it.

Dually, one may interpret the vertices of this graph as the points; then each edge indicates that the corresponding points are collinear. Systematically, this graph is known as the *point graph* or *collinearity graph*.

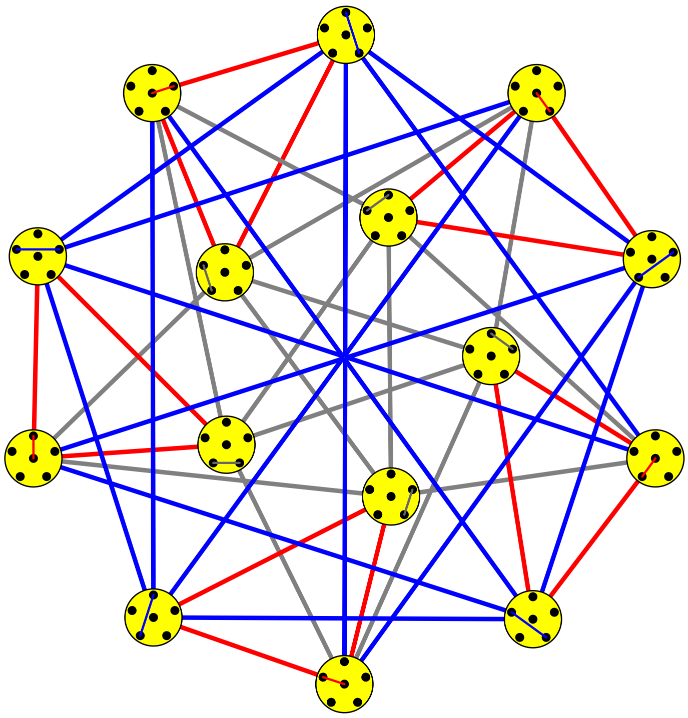


Figure 9: The line graph of the doily, showing collinearity.

7.1 Remark. Two transpositions in Sym(6) commute if, and only if, the corresponding vertices in the line graph are joined by an edge.

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