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THE MINIMAL GENUS PROBLEM FOR ELLIPTIC SURFACES

ABSTRACT. We partly solve the minimal genus problem for embedded surfaces in the case of elliptic 4-manifolds. This involves a certain restricted transitivity property of the action of the orientation preserving diffeomorphism group on the second homology. In all cases we consider we get the minimal possible genus allowed by the adjunction inequality.

1. INTRODUCTION

Starting with the classical work of Kervaire and Milnor [10], who showed that certain second homology classes in simply-connected 4-manifolds are not represented by embedded spheres, the question arose to find for a given homology class in a 4-manifold the minimal genus of an embedded closed connected oriented surface realizing that class. This question has been solved at least partly for rational and ruled surfaces and for 4-manifolds with a free circle action [4, 5, 14, 15, 16, 17, 18, 19, 25]. On symplectic 4-manifolds the question is related to the Thom conjecture [12, 22, 23]. In particular, the adjunction inequality from Seiberg-Witten theory gives a lower bound on the genus of a surface representing a homology class in a closed oriented 4-manifold with a basic class and we can then ask if this lower bound is indeed realized. Usually the question is more tractable for classes of positive self-intersection and is still open in most situations in the case of negative self-intersections. In particular, it is still unknown whether there exist embedded spheres in the $K3$ surface of arbitrarily negative self-intersection.

Another interesting class of 4-manifolds are elliptic surfaces. We will restrict to (relatively) minimal simply-connected elliptic surfaces with $b_2^+ > 1$, but generalizations should be possible. Note that every orientation preserving diffeomorphism of a closed, oriented 4-manifold induces an isometry of the intersection form on the second homology (modulo torsion). A very useful fact is that for elliptic surfaces the image of the orientation preserving diffeomorphism group in the orthogonal group of the intersection form is known. This is due to Borcea, Donaldson and Matumoto [1, 2, 21] for the $K3$ surface and to Friedman-Morgan and Lönne in the general case [6, 20]. We will combine this knowledge with the work of Wall on the transitivity of the orthogonal groups of unimodular quadratic forms [26]. Similar to the case of rational surfaces, this will allow us to reduce the problem of representing a homology class by a minimal genus surface to certain special classes. We cannot treat the minimal genus problem in full generality. Instead we will concentrate on the first interesting special cases that come to mind. To state one of the results, we will prove the following in the special case that the elliptic surface has no multiple fibres, i.e. is given by a surface $E(n)$ with $n \geq 2$:

Theorem 1.1. *Let X be an elliptic surface diffeomorphic to $E(n)$ with $n \geq 2$. Suppose A is a class in $H_2(X;\mathbb{Z})$ orthogonal to the canonical class K and of self-intersection $A^2 = 2c-2$ with $c \geq 0$. Then A is represented by a surface of genus c in X. This is the minimal possible genus if A is non-zero.*

There is similar, slightly more restrictive theorem in the case of elliptic surfaces with multiple fibres. We are also interested if we can realize homology classes by surfaces that are contained in

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certain nice neighbourhoods inside the elliptic surface. The neighbourhood we consider is the Gompf nucleus $N(2)$.

Notations. In the following X will denote a relatively minimal simply-connected elliptic surface with the complex orientation. By an elliptic surface we always mean a surface of this kind. Using the classification of elliptic surfaces [9] X is diffeomorphic to $E(n)_{p,q}$, where the coprime indices denote logarithmic transformations. We restrict to the case $n \geq 2$ or equivalently $b_2^+ > 1$; see [14] for a discussion of Dolgachev surfaces $E(1)_{p,q}$ with $b_2^+ = 1$. All self-diffeomorphisms of X are orientation preserving. We often denote a closed oriented surface in X and the homology class it represents by the same symbol.

2. ACTION OF THE DIFFEOMORPHISM GROUP

Let $H_2(X)$ denote the integral second homology of X and Diff$^+(X)$ the group of orientation preserving self-diffeomorphisms of X. The intersection form on second homology induces a unimodular quadratic form on $H_2(X)$. We denote by O the orthogonal group of all automorphisms of $H_2(X)$ that preserve the intersection form. The elements of this group are called automorphisms of the intersection form. The action of diffeomorphisms on homology defines a group homomorphism Diff$^+(X) \to O$.

There is a way to choose an orientation on all maximal positive definite linear subspaces of $H_2(X;\mathbb{R})$, cf. [24]: Fix any such subspace U_0 and let $\pi\colon H_2(X;\mathbb{R}) \to U_0$ denote the orthogonal projection. The restriction of π to any maximal positive definite subspace U is an isomorphism with U_0. Choosing an orientation for U_0 we get an orientation for all maximal positive definite subspaces U via π. This orientation varies continuously with U. The *spinor norm* of an element $\phi \in O$ is defined to be ± 1 depending on whether ϕ preserves or reverses the orientation on a maximal positive definite subspace of $H_2(X;\mathbb{R})$. A deformation argument shows that this does not depend on the choice of such a subspace. The subgroup of O of elements of spinor norm 1 is denoted by O'.

Definition 2.1. We let K denote the canonical class of X, which is minus the first Chern class. If X is not the $K3$ surface let k denote the Poincaré dual of K divided by its divisibility. If X is the $K3$ surface let k denote the class of a general fibre. In any case, k is a primitive class of self-intersection zero. We also choose a second homology class V such that $k \cdot V = 1$. For example if X has no multiple fibres we can choose for V a section of an elliptic fibration. We denote by O_k the automorphisms fixing k and by O'_k those of spinor norm 1.

The following was proved in [20].

Theorem 2.2. *The image of the diffeomorphism group Diff$^+(X)$ in O is equal to O' for the $K3$ surface and contains O'_k for all other elliptic surfaces X.*

We now consider integral unimodular quadratic forms in general. We let H denote the even hyperbolic form of rank 2 and E_8 the standard positive definite even form of rank 8. A *standard basis* for H is a basis e, f such that

$$e^2 = 0, f^2 = 0, e \cdot f = 1.$$

Let Q denote the quadratic form $Q = lH \oplus m(-E_8)$ with $l \geq 2$ and $m \in \mathbb{Z}$. In [26] Wall proved the following.

Theorem 2.3. *The orthogonal group of Q acts transitively on primitive elements of given square.*

We want to deduce the following.

Proposition 2.4. *The subgroup of elements of spinor norm 1 in the orthogonal group of Q acts transitively on primitive elements of given square.*

We first prove the following lemma.

Lemma 2.5. *For any even number $2a$ there exist primitive elements p and q of square $2a$ and automorphisms of Q of spinor norm $+1$ and -1 which map p to q.*

Proof. We consider $Q = lH \oplus m(-E_8)$ and let e, f denote a standard basis for the first H summand. Let $p = e + af$ and $q = -e - af$. Then $p^2 = q^2 = 2a$. Consider the automorphism of Q which is minus the identity on the first H summand and the identity on all other summands and the automorphism which is minus the identity on the first two H summands and the identity on all other summands. These automorphisms have spinor norm -1 and $+1$ and map p to q. □

We now prove Proposition 2.4.

Proof. Let x and y be arbitrary primitive elements of square $2a$ and let p and q be the elements from the lemma of the same square. By Wall's theorem there exist automorphisms in O mapping x to p and q to y. Choosing an automorphism that maps p to q of the correct spinor norm we get by composing an automorphism of spinor norm $+1$ mapping x to y. □

We now consider the elliptic surface X.

Lemma 2.6. *The self-intersection number V^2 is even if and only if X is spin.*

Proof. The intersection form on the span of k and V is unimodular, hence it is unimodular on the orthogonal complement. The intersection form on this complement is even, since the canonical class K is characteristic. The claim now follows because X is spin if and only if the intersection form on both summands is even. □

Let $V^2 = 2a$ in the spin case and $V^2 = 2a + 1$ in the non-spin case.

Definition 2.7. Define an element $W = V - ak$. Then the intersection form on the span of k and W is the form H in the spin case and the form H' given by

$$H' = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

in the non-spin case. Note that H' is isomorphic to $\langle +1 \rangle \oplus \langle -1 \rangle$.

The complete intersection form of X is then given by

$$H \oplus lH \oplus m(-E_8) \text{ or } H' \oplus lH \oplus m(-E_8), \tag{2.1}$$

where $l \geq 2$ since $b_2^+ \geq 3$. We also want to choose a standard basis for the second H summand: The $K3$ surface is known to contain a rim torus R of self-intersection zero and a vanishing sphere S of self-intersection -2 such that R and S intersect transversely in one positive point. Both arise from the fibre sum construction $K3 = E(1)\#_{F=F}E(1)$ along a general fibre F. See Section 4 for an explicit model of the rim torus; the vanishing sphere is obtained by sewing together two vanishing disks of relative self-intersection -1 coming from elliptic Lefschetz fibrations on $E(1)$. Recall the following definition from [7]:

Definition 2.8. The *nucleus* $N(2)$ is defined as the 4-manifold with boundary given by the neighbourhood of a cusp fibre and a section in $K3$.

Note that the nucleus contains also a smooth torus homologous to the cusp. In addition to the nucleus given by the definition, the $K3$ surface contains two other embedded copies of $N(2)$, disjoint from the first one. The rim torus R and the vanishing sphere S are embedded in one such copy [8]. Since this nucleus is disjoint from a general fibre it is still contained in an arbitrary elliptic surface X of the type above because the fibre sums and the logarithmic transformations resulting in the manifold $X = E(n)_{p,q}$ are performed in the complement of the nucleus. We also choose the surface representing the class V to be disjoint from this nucleus.

Definition 2.9. Let T denote the torus of self-intersection zero obtained by smoothing the intersection between R and S. Then T represents the class $R+S$ and the classes R and T are a standard basis for the second H summand in the intersection form of the elliptic surface X.

Using Theorem 2.2 and Proposition 2.4 we have the following:

Proposition 2.10. *Let $A \in H_2(X)$ be an arbitrary element. Then there exists a self-diffeomorphism of X which maps A to*

$$A' = \alpha k + \beta W + \gamma R + \delta T,$$

where $\alpha, \beta, \gamma, \delta$ are certain integers. The diffeomorphism is the identity on the span of k and W. Suppose X is the $K3$ surface and the class A primitive. Then we can map A via a self-diffeomorphism to

$$A' = \alpha R + S,$$

where $2\alpha - 2 = A^2$. Hence the self-diffeomorphisms of the $K3$ surface act transitively on primitive elements of given square.

The result for the $K3$ surface is well-known [11, 14]. As a final preparation, recall the following theorem [9], containing the so-called *adjunction inequality*:

Theorem 2.11. *Let Y be a closed oriented 4-manifold with $b_2^+ > 1$. Assume that Σ is an embedded oriented connected surface in Y of genus $g(\Sigma)$ with self-intersection $\Sigma^2 \geq 0$, such that the class represented by Σ is non-zero. Then for every Seiberg-Witten basic class L we have*

$$2g(\Sigma) - 2 \geq \Sigma \cdot \Sigma + |L \cdot \Sigma|.$$

If Y is of simple type and $g(\Sigma) > 0$, then the same inequality holds for $\Sigma \subset Y$ with arbitrary square $\Sigma \cdot \Sigma$.

Note that if L is a basic class, i.e. a characteristic class in $H^2(Y;\mathbb{Z})$ with non-vanishing Seiberg-Witten invariant, then $-L$ is also a basic class. The basic classes of the elliptic surfaces $X = E(n)_{p,q}$ are completely known [3]. They are given by the set

$$\{rk \mid r \equiv npq - p - q \bmod 2,\ |r| \leq npq - p - q\}.$$

The basic classes are multiples of the class k where the maximal values at the end are given (up to sign) by the canonical class K of the elliptic surface X. Hence the adjunction inequality for the elliptic surfaces X reduces to the statement that

$$2g(\Sigma) - 2 \geq \Sigma \cdot \Sigma + |K \cdot \Sigma|.$$

3. Minimal genus problem for the $K3$ surface

The minimal genus problem for classes of non-negative square in the $K3$ surface has already been solved [14] using the $K3$ case of Proposition 2.10. We want to recall this solution and also show that we can realize these surfaces in a nucleus $N(2)$ in a certain standard way. Note that the adjunction inequality for the $K3$ surface implies because of $K = 0$ for the genus of a smooth surface Σ that $2g(\Sigma) - 2 \geq \Sigma \cdot \Sigma$ if the homology class represented by this surface is non-zero.

Definition 3.1. By *the standard surface of genus g embedded in the nucleus $N(2)$* we mean the section of self-intersection -2 ($g = 0$), the general fibre of self-intersection 0 ($g = 1$) or the surface of genus $g \geq 2$ and self-intersection $2g - 2$ obtained by smoothing the intersection points of the section and g parallel copies of the general fibre.

According to Proposition 2.10 we can map any primitive class in the $K3$ surface via a self-diffeomorphism to a class in the nucleus. Hence we get:

Corollary 3.2. *Consider the $K3$ surface. Every primitive class of self-intersection $2c - 2$ with $c \geq 0$ is represented by a surface of genus c. We can assume that it is embedded as the standard surface in a nucleus $N(2)$ inside $K3$. This is the minimal possible genus.*

To solve the case of divisible classes with non-negative square we use Lemma 7.7 in [13] due to Kronheimer-Mrowka (see also Lemma 14 in [14]):

Lemma 3.3. *Let Y be a closed connected oriented 4-manifold. Let $a(\Sigma) = 2g(\Sigma) - 2 - \Sigma \cdot \Sigma$. If $h \in H_2(Y)$ is a homology class with $h \cdot h \geq 0$ and Σ_h is a surface of genus g representing h and $g \geq 1$ when $h \cdot h = 0$, then for all $r > 0$, the class rh can be represented by an embedded surface Σ_{rh} with*

$$a(\Sigma_{rh}) = ra(\Sigma_h).$$

In particular, we can apply the construction of this lemma to divisible classes of non-negative square inside the nucleus $N(2)$ to get surfaces that represent these classes in the nucleus (the construction in the proof of this lemma works in a tubular neighbourhood of Σ_h and does not need the assumption that Y is closed). In this case $a(\Sigma_h)$ is zero, hence also $a(\Sigma_{rh})$ is zero. We have:

Corollary 3.4. *Every class in $H_2(N(2))$, not necessarily primitive, which has self-intersection $2c-2$ with $c \geq 0$ is represented by an embedded surface of genus c in $N(2)$.*

We call these surfaces in the nucleus *standard*. The transitivity of the action of the diffeomorphism group then implies that every divisible class of non-negative square in $K3$ can also be represented by such a standard surface inside a nucleus $N(2)$. Hence Corollary 3.2 holds without the assumption that the class is primitive.

4. Minimal genus problem for other elliptic surfaces

We now consider the general case of relatively minimal simply-connected elliptic surfaces X with $b_2^+ > 1$. Note that the adjunction inequality implies for surfaces Σ orthogonal to K again that $2g(\Sigma) - 2 \geq \Sigma \cdot \Sigma$. The self-intersection of such a surface is even. Using Proposition 2.10 and Corollary 3.4 we get:

Corollary 4.1. *Let X be an elliptic surface. Then every class A of self-intersection $2c - 2$ with $c \geq 0$ that is orthogonal to the classes K and V is represented by a surface of genus c. We can assume that it is embedded as the standard surface in a nucleus $N(2)$ in the 4-manifold X. This is the minimal possible genus if the class is non-zero.*

Proof. The assumptions imply that A can be mapped via a diffeomorphism to $A' = \gamma R + \delta S$. Since R and S are constructed in a nucleus $N(2)$ the claim follows. □

Remark 4.2. If we relax the assumption and only assume that A is orthogonal to K it seems that the surface is in general not contained in a nucleus $N(2)$. For example the general fibre is contained in a nucleus $N(n)_{p,q}$.

We can deal with the case $A^2 = -2$ in a slightly more general situation:

Proposition 4.3. *Let X be an elliptic surface. Then any class A orthogonal to K and of self-intersection -2 is represented by the standard sphere in a nucleus $N(2)$ in the 4-manifold X.*

Proof. The assumptions imply that there exists a self-diffeomorphism of X mapping A to

$$A' = \alpha k + S,$$

where S is the vanishing sphere. Consider the following map ϕ on $H_2(X)$ which on the first two summands of the intersection form is given by

$$\begin{aligned} k &\mapsto k \\ W &\mapsto W + \alpha R \\ R &\mapsto R \\ S &\mapsto S - \alpha k \end{aligned}$$

and is the identity on all other summands. It is easy to check that ϕ is an isometry. Letting α be a real number and taking $\alpha \to 0$ we see that ϕ has spinor norm $+1$. Hence it is an element in O'_k and therefore induced by a self-diffeomorphism. It maps A' to S. This implies the claim. □

Remark 4.4. This result should be compared to the fact that every class of square -2 in the complement of a general fibre in X is represented by an embedded sphere [6, 20].

We now restrict to the case of elliptic surfaces without multiple fibres, i.e. $X = E(n)$, because the following arguments seem to work only in this case. The class k is represented by a general fibre F. We also have the rim torus R. Proposition 2.10 implies:

Lemma 4.5. *If A is a class orthogonal to K and of self-intersection zero then there exists a self-diffeomorphism of X that maps A to*

$$A' = \alpha F + \gamma R.$$

We want to show that A' can be represented by an embedded torus. The construction involves the circle sum from [19]. The idea is the following: Let Σ_0 and Σ_1 denote two disjoint connected embedded oriented surfaces in a 4-manifold Y. We can tube them together in the standard way to get a surface of genus $g(\Sigma_0) + g(\Sigma_1)$. Sometimes, however, we can perform a different surgery that results in a surface of smaller genus. Let $S^1_i \subset \Sigma_i$ denote embedded circles that represent non-trivial homology classes in the surfaces. In each surface we delete an annulus $S^1_i \times I$. We get two disjoint surfaces whose boundaries consist of two circles for each surface. We want to connect these circles by annuli embedded in Y. There are several ways to do this: One possibility is to connect the circles from the same surface. In this way we simply get back the surfaces Σ_0 and Σ_1. Another possibility is to connect the boundary circles from different surfaces. If this is possible we get an embedded connected surface of genus $g(\Sigma_0) + g(\Sigma_1) - 1$ representing the class $[\Sigma_0] + [\Sigma_1]$.

The construction works if we can find an embedded annulus Δ in Y that intersects the surfaces Σ_0 and Σ_1 precisely in the circles S^1_0 and S^1_1. We also need a nowhere vanishing normal vector field along Δ that at the ends of Δ is tangential to the surfaces Σ_0 and Σ_1. The annuli connecting the four boundary circles are then constructed as normal push-offs of the annulus Δ.

Lemma 4.6. *There exists an embedded annulus Δ connecting the tori F and R that satisfies the necessary assumptions for the circle sum in* [19].

Proof. The elliptic surface $X = E(n)$ is obtained as a fibre sum of $E(n-1)$ and $E(1)$ along a general fibre. Let $S^1 \times S^1 \times D^2$ denote a tubular neighbourhood of the fibre in one of the summands. We think of D^2 as the unit disk in the complex plane and let I denote the interval $[\frac{1}{2}, 1]$ along the real axis. In forming the fibre sum we delete the open tubular neighbourhood of radius $\frac{1}{4}$ of the general fibre in the centre of the tubular neighbourhood. The fibre F in X is realized as $S^1 \times S^1 \times \{\frac{1}{2}\}$ while the rim torus R is $S^1 \times \{*\} \times \partial D^2$. Consider the annulus $\Delta = S^1 \times \{*\} \times I$. It intersects the tori F and R precisely in the circles $S^1_F = S^1 \times \{*\} \times \{\frac{1}{2}\}$ and $S^1_R = S^1 \times \{*\} \times \{1\}$. Let v_F be a unit tangent vector to S^1 in the point $*$ and v_R a unit tangent vector to ∂D^2 in 1. Then

$$e_F = S^1 \times v_F \times \{\tfrac{1}{2}\}$$

and

$$e_R = S^1 \times \{*\} \times v_R$$

are framings of the circles S^1_F and S^1_R inside the tori. Consider the normal vector field along the annulus Δ given on $S^1 \times \{*\} \times t$ by

$$e = S^1 \times (2-2t)v_F \times t \times (2t-1)v_R.$$

This is equal to the framings e_F and e_R on the boundary and is the required framing of the annulus. □

This construction allows us to circle sum F and R. A similar, but easier construction allows us to circle sum $|\alpha|$ parallel copies of F and $|\gamma|$ parallel copies of R with a suitable orientation to get embedded tori Σ_0 and Σ_1 representing the classes αF and γR. The torus Σ_0 contains as an open subset a copy of the torus F with an annulus deleted, and similarly for Σ_1. Circle summing Σ_0 and Σ_1 along these subsets we get an embedded torus representing the class $\alpha F + \gamma R$. This construction proves:

Theorem 4.7. *Let X be an elliptic surface without multiple fibres. Then any class A orthogonal to K and of self-intersection zero is represented by an embedded torus.*

This is clearly the minimal possible genus allowed by the adjunction inequality if the class A is non-zero. The same method can be used to prove the following generalization:

Theorem 4.8. *Let X be an elliptic surface without multiple fibres. Suppose A is a class orthogonal to K such that $A^2 = 2c-2$ with $c \geq 0$. Then A is represented by a surface of genus c in X. This is the minimal possible genus if A is non-zero.*

Proof. The cases $c = 0$ and $c = 1$ have been proved above. We can assume that $c \geq 2$. The assumptions imply that there exists a self-diffeomorphism of X mapping A to

$$A' = \alpha F + \gamma R + \delta T,$$

where γ and δ are positive with $\gamma\delta = c-1$. We circle sum $|\alpha|$ parallel copies of F with a suitable orientation to get a torus Σ_0 representing αF. Taking circle sums of parallel copies of the tori R and T we get tori representing γR and δT that intersect transversely in $\gamma\delta$ points. Smoothing these intersections we get a surface Σ_1 of genus $\gamma\delta + 1 = c$. This surface contains as an open subset a copy of the torus R with an annulus and δ points deleted. We circle sum the surface Σ_1 to the torus Σ_0 to get an embedded surface of genus c representing A'. □

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