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Fachbereich Mathematik
Fakultät Mathematik und Physik
Universität Stuttgart
Pfaffenwaldring 57
D-70 569 Stuttgart

E-Mail: preprints@mathematik.uni-stuttgart.de
WWW: http://www.mathematik.uni-stuttgart.de/preprints

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Rational conjugacy of torsion units in integral group rings of non-solvable groups

Andreas Bächle, Leo Margolis

Abstract. We introduce a new method to study rational conjugacy of torsion units in integral group rings. We use this method to prove the first Zassenhaus Conjecture for $\mathrm{PSL}(2,19)$. We then use the standard HeLP-method to prove the Zassenhaus Conjecture for $\mathrm{PSL}(2,23)$ and the introduced method to show that there are no units of order 6 in the normalized units of the integral group rings of the groups M_{10} and $\mathrm{PGL}(2,9)$. This last fact completes the proof of a theorem of W. Kimmerle and A. Konovalov that the prime graph of a group G coincides with the prime graph of the corresponding group of normalized units of the integral group ring, if the order of G is divisible by at most three primes.

Let G be a finite group, $\mathbb{Z}G$ the integral group ring of G and $\mathrm{V}(\mathbb{Z}G)$ the group of augmentation one units in $\mathbb{Z}G$. The most famous open conjecture regarding torsion units in $\mathbb{Z}G$ is

The Zassenhaus Conjecture (ZC): Let $u \in \mathrm{V}(\mathbb{Z}G)$ be a torsion unit. Then there exist a unit $x \in \mathbb{Q}G$ and $g \in G$ such that $x^{-1}ux = g$.

If for a unit u such x and g exist we say that u is rationally conjugate to g. Though the study of the Zassenhaus Conjecture mostly concentrated around solvable groups (e.g.

A. Weiss proved it for nilpotent groups [Wei91], see [Her06], [Her08a], or [CMdR13] for more recent results), it was also sometimes examined for non-solvable groups. E.g. it is known for A_5 [LP89], S_5 [LT91], A_6 [Her08b], or $\mathrm{PSL}(2, p)$ for $p \leq 17$ a prime [Her07], [KK12], [Gil12]. Sometimes weaker version of ZC are also considered, such as

The Prime Graph Question (PQ): Let p and q be different primes such that $\mathrm{V}(\mathbb{Z}G)$ has a unit of order pq. Does this imply that G has an element of that order?

This is the same as to ask, whether G and $\mathrm{V}(\mathbb{Z}G)$ have the same prime graph. Much more is known here: E.g. it has an affirmative answer for solvable groups [Kim06] or the series $\mathrm{PSL}(2, p)$, p a prime [Her07]. V. Bovdi, A. Konovalov, and others also proved it for many sporadic simple groups, see e.g. [BKS07], [BJK11], [BK12]. W. Kimmerle and A. Konovalov proved, that (PQ) holds for groups whose order is divisible by at most three primes, if there are no units of order 6 in $\mathrm{V}(\mathbb{Z}M_{10})$ and $\mathrm{V}(\mathbb{Z}\,\mathrm{PGL}(2, 9))$ [KK12].
All proofs of ZC for non-solvable groups rely on the so called Luthar-Passi-Hertweck-method [LP89], [Her07], sometimes referred to as the HeLP-method. But in many cases this method does not suffice to prove ZC, e.g. it fails for A_6 [Her08b], $\mathrm{PSL}(2, 19)$ (see below) or M_{11} [BK07]. Sometimes special arguments were considered in such situations as in [LT91], [Her06, Ex. 2.6], [Her08b]. But these arguments were designed for very special situations and are either hard to generalize, such as the argument in [Her08b], or seem not to give new information in other situations, such as the argument in [Her06, Ex. 2.6].

In this paper we introduce a new method to study rational conjugacy of torsion units inspired by M. Hertweck's arguments for proving ZC for the alternating group of degree 6 [Her08b]. This method is especially interesting for units of mixed order (i.e. not of prime power order) and in combination with the HeLP-method. We then give two applications of this method to prove: (ZC for $\mathrm{PSL}(2, 23)$ is proved using known methods.)

Theorem 1. *The Zassenhaus Conjecture holds for the groups* $\mathrm{PSL}(2, 19)$ *and* $\mathrm{PSL}(2, 23)$.

Theorem 2. *There are no units of order 6 in* $\mathrm{V}(\mathbb{Z}M_{10})$ *and in* $\mathrm{V}(\mathbb{Z}\,\mathrm{PGL}(2, 9))$. *Here* M_{10} *denotes the Mathieu group of degree 10.*

Theorem 2 together with [KK12, Th. 2.1, Th. 3.1] directly yields:

Corollary 3. *Let* G *be a group, whose order is divisible by at most three primes. Then the prime graph question has a positive answer for* G.

1 From eigenvalues under ordinary representations to the modular module structure

Let G be a finite group. The main tool to study rational conjugacy of torsion units are partial augmentations: Let $u = \sum_{g \in G} a_g g \in \mathbb{Z}G$ and x^G be the conjugacy class of the element $x \in G$ in G. Then $\varepsilon_x(u) = \sum_{g \in x^G} a_g$ is called the partial augmentation of u at x. This relates to ZC via:

Lemma 1.1 ([MRSW87, Th. 2.5]). *Let $u \in \mathrm{V}(\mathbb{Z}G)$ be a torsion unit. Then u is rationally conjugate to a group element if and only if $\varepsilon_x(u^k) \geq 0$ for all $x \in G$ and all powers u^k of u.*

It is well known that if $u \neq 1$ is a torsion unit in $\mathrm{V}(\mathbb{Z}G)$, then $\varepsilon_1(u) = 0$ by the so-called Berman-Higman Theorem [Seh93, Prop. 1.4]. If $\varepsilon_x(u) \neq 0$, then the order of x divides the order of u [MRSW87, Th. 2.7], [Her06, Prop. 3.1]. Moreover the exponents of G and of $\mathrm{V}(\mathbb{Z}G)$ coincide [CL65]. We will use this in the following without further mentioning.

Let K be a field and D a K-representation of G with corresponding character χ. If χ and all partial augmentations of u and all its powers are known and the characteristic of K does not divide the order of u we can compute the eigenvalues of $D(u)$ in the algebraic closure of K (there will be plenty of examples in §2). Let n be the order of u. The HeLP-method makes use of the fact, that the multiplicity of each n-th root of unity as an eigenvalue of $D(u)$ is a non-negative integer.

Notations: We will use the following notation: p will always denote a prime, $\mathbb{Q}_p$ the p-adic completion of $\mathbb{Q}$ and $\mathbb{Z}_p$ the ring of integers of $\mathbb{Q}_p$. By R we denote a complete local ring with maximal ideal P containing p. The field of fractions of R will be denoted by K and the residue class field of R by k. The reduction modulo P, also with respect to modules, will be denoted by $\bar{\ }$.

The idea of our method is, that if D is an R-representation of a group G and u is a torsion unit in $\mathbb{Z}G$ of order divisible by p, we can reduce D modulo P and obtain restrictions on the isomorphism type of kG-modules as $k\langle \bar{u} \rangle$-modules. The connections between the eigenvalues of ordinary representations and the isomorphism type of the modular modules for some cases are contained in the following propositions which are easy consequences of known modular and integral representation theory.

The first proposition is standard knowledge in modular representation theory and may be found e.g. in [HB82, Th. 5.3, Th. 5.5].

Proposition 1.2. *Let $G = \langle g \rangle$ be a cyclic group of order $p^a m$, where p does not divide m. Let k be a field of characteristic p containing a primitive m-th root of unity ζ. Then*

a) Up to isomorphism there are m simple kG-modules. All these modules are one-dimensional as k-vector spaces, g^m acts trivially on them and g^{p^a} acts as ζ^i for $1 \leq i \leq m$. We call this modules k_1, ..., k_m.

b) The projective indecomposable kG-modules are of dimension p^a. They are all uniserial and all composition factors of a projective indecomposable kG-module are isomorphic. There are m projective indecomposable kG-modules.

c) Each indecomposable kG-module is isomorphic to a submodule of a projective indecomposable module. So there are $p^a m$ indecomposable modules, which are all uniserial and all composition factors of an indecomposable kG-module are isomorphic.

Using Proposition 1.2 and the fact that idempotents may be lifted [CR81, Th. 30.4] we obtain:

Proposition 1.3. *Let $G = \langle g \rangle$ be a cyclic group of order $p^a m$, where p does not divide m. Let R be a complete local ring containing a primitive m-th root of unity ζ. Let D be an R-representation of G and let L be an RG-lattice affording this representation. Let A_i be sets with multiplicities of p^a-th roots of unity such that $\zeta A_1 \cup \zeta^2 A_2 \cup ... \cup \zeta^m A_m$ are the complex eigenvalues of $D(g)$. (We may have $A_i = \emptyset$.) Let V_1, ..., V_m be KG-modules such that if E_i is a representation of G affoding V_i the eigenvalues of $E_i(g)$ are $\zeta^i A_i$.*
Then $L \cong L_1 \oplus ... \oplus L_m$ and $\bar{L} \cong \bar{L}_1 \oplus ... \oplus \bar{L}_m$ such that $\mathrm{rank}_R(L_i) = \dim_k(\bar{L}_i) = |A_i|$. Moreover $K \otimes_R L_i \cong V_i$ and the only composition factor of $\bar{L}_i$ is k_i. (See notation in Proposition 1.2.)

In some situations we can give a full description of the L_i depending on A_i. The easiest one is recorded in the next proposition. It is a consequence of [HR62, Th. 2.6].

Proposition 1.4. *Let the notation be as in Proposition 1.3, assume* $|G| = p$ *and that* K *is unramified over* $\mathbb{Q}_p$. *Let* ξ *be a primitive* p*-th root of unity. Up to isomorphism there are 3 indecomposable* RG*-lattices* M_1, M_2, M_3. *Each* $\bar{M}_i$ *remains indecomposable. The* R*-rank and the corresponding eigenvalues of* $D(g)$ *are:* $\text{rank}_R(M_1) = 1$ *with eigenvalue* 1, $\text{rank}_R(M_2) = p - 1$ *with eigenvalues* $\xi, \xi^2, ..., \xi^{p-1}$, *and* $\text{rank}_R(M_3) = p$ *with eigenvalues* $1, \xi, \xi^2, ..., \xi^{p-1}$.

Notation: We denote the lattices from Proposition 1.4 with the natural names $M_1 = R$, $M_2 = I(RC_p)$, and $M_3 = RC_p$.

Remark 1.5. Let the notation be as in Proposition 1.3. Some other useful results are:

a) The Krull-Schmidt-Azumaya Theorem holds for L [CR81, Th. 30.6].

b) Assume $a = 1$ and L_i is indecomposable such that $K \otimes_R L_i \cong a_1 S_1 \oplus a_2 S_2$ with simple non-isomorphic KG-modules S_1 and S_2 and $a_1, a_2 \in \mathbb{N}_0$. Then $a_1, a_2 \leq 1$ [Gud67, Th. 2.2].

c) Assume that up to isomorphism there exist exactly 3 simple $K\langle g^m \rangle$-modules S_1, S_2 and S_3. Let S_1 be the trivial module. If L_i is indecomposable and $a_1, a_2, a_3 \in \mathbb{N}_0$ are such that $K \otimes_R L_i \cong a_1 S_1 \oplus a_2 S_2 \oplus a_3 S_3$, then $a_1 \leq 2$ and $a_2, a_3 \leq 1$ [Jac67, Prop. 8].

2 Applications

For a group G we denote by χ_i an ordinary character of G and by D_i a representation of G affording this character. By φ_i we denote a Brauer character and by Θ_i a representation affording φ_i. We write $D_i(u) \sim \text{diag}(\alpha_1, ..., \alpha_j)$ or $\Theta_i(u) \sim \text{diag}(\alpha_1, ..., \alpha_j)$ to indicate that $\alpha_1, ..., \alpha_j$ are the eigenvalues (with multiplicities) of the corresponding matrix. By ζ_n we will denote some fixed complex primitive n-th root of unity.
Let K be an algebraically closed field, D a K-representation of G with character χ and u a torsion unit in $\text{V}(\mathbb{Z}G)$ such that the characteristic of K does not divide the order of u. Let m and n be natural numbers such that $u^{m+n} = u$. Let $D(u^m) \sim \text{diag}(\alpha_1, ..., \alpha_k)$ and $D(u^n) \sim \text{diag}(\beta_1, ..., \beta_k)$. As $D(u^m)$ and $D(u^n)$ are simultaneously diagonalizable over K

this means $D(u) \sim \mathrm{diag}(\alpha_1\beta_{i_1}, ..., \alpha_k\beta_{i_k})$ with $\{i_1, ..., i_k\} = \{1, ..., k\}$. On the other hand $\chi(u) = \sum_{x^G} \varepsilon_x(u)\chi(x)$, where the sum runs over all conjugacy classes x^G of G. Comparing this two computations is the basic idea of the Luthar-Passi method. We will use it freely in the following computations.

2.1 The groups $\mathrm{PSL}(2,p)$: Proof of Theorem 1

Rational conjugacy of torsion units in integral group rings over the groups $\mathrm{PSL}(2,q)$ were studied by Hertweck in [Her07]. For the rest of the paragraph let p be a prime. Combining some propositions from that note we directly obtain:

Proposition 2.1 (Hertweck). *Let $G = \mathrm{PSL}(2,p)$ and u a torsion unit in $\mathrm{V}(\mathbb{Z}G)$. Then there is an element $g \in G$ of the same order as u. Moreover if u is of prime order or of order 6 then u is rationally conjugate to a group element.*

Proof. [Her07, Proposition 6.1, Proposition 6.3, Proposition 6.4, Proposition 6.6, and Proposition 6.7].

The HeLP-method verifies the Zassenhaus-Conjecture for $\mathrm{PSL}(2,p)$ if $p \leq 17$. We give a quick account: ZC is solved for $p \in \{2,3\}$ already in [SW86], $p = 5$ in [LP89], $p = 7$ in [Her06], $p \in \{11,13\}$ in [Her07], and $p = 17$ independently in [KK12] and [Gil12]. The HeLP-method also suffices to prove ZC for $p = 23$ (see below), but not for $p = 19$. We will always use the character tables respectively Brauer tables from the ATLAS [Wil] [1].
For $G = \mathrm{PSL}(2,p)$ and $p > 2$ we have $|\,\mathrm{PSL}(2,p)| = \frac{(p-1)p(p+1)}{2}$, there are cyclic subgroups of order $\frac{p-1}{2}$, p, and $\frac{p+1}{2}$ in G and every cyclic subgroup of G lies in a conjugate of such a subgroup. Elements of order p lie in exactly two conjugacy classes and if g is an element of p'-order the only conjugate of g in $\langle g \rangle$ is g^{-1}. All this follows from a result of Dickson [Hup67, Satz 8.27].

Proof of Theorem 1. Proof of ZC for $\mathrm{PSL}(2,19)$: We give the parts of the character tables relevant for our proof in the tables 1, 2, and 3.

[1] This tables are accessible in `GAP` [GAP12] via the commands `Display(CharacterTable("PSL(2,p)"));` and `Display(CharacterTable("PSL(2,p)") mod p);`.

	1a	2a	5a	5b	10a
χ_{18}	18	-2	$-\alpha$	$-\beta$	$-\alpha$
χ_{19}	19	-1	-1	-1	-1

with $\alpha = \zeta_5 + \zeta_5^4, \quad \beta = \zeta_5^2 + \zeta_5^3$

Table 1: Part of the ordinary character table of PSL(2, 19)

	1a	2a	3a	5a	5b	9a	9b	9c	10a	10b
φ_3	3	-1	$\cdot$	$-\beta$	$-\alpha$	x	y	z	$-2\alpha-\beta$	$-\alpha-2\beta$
φ_5	5	1	-1	$\cdot$	$\cdot$	x'	y'	z'	-2α	-2β

with
$\alpha = \zeta_5 + \zeta_5^4$, $\beta = \zeta_5^2 + \zeta_5^3$,
$x = -\zeta_9^3 + \zeta_9^4 + \zeta_9^5 - \zeta_9^6$, $x' = -\zeta_9^2 - \zeta_9^3 - \zeta_9^6 - \zeta_9^7$,
$y = -\zeta_9^2 - \zeta_9^3 - \zeta_9^4 - \zeta_9^5 - \zeta_9^6 - \zeta_9^7$, $y' = -\zeta_9^3 - \zeta_9^4 - \zeta_9^5 - \zeta_9^6$
$z = \zeta_9^2 - \zeta_9^3 - \zeta_9^6 + \zeta_9^7$, $z' = \zeta_9^2 - \zeta_9^3 + \zeta_9^4 + \zeta_9^5 - \zeta_9^6 + \zeta_9^7$.

Table 2: Part of the Brauer table of PSL(2, 19) and $p = 19$

By Proposition 2.1 only normalized units of order 9 and 10 have to be checked. Let u be of order 9. By [Her07, Prop. 6.5] we have $\varepsilon_{3a}(u) = 0$, thus $\varepsilon_{9a}(u) + \varepsilon_{9b}(u) + \varepsilon_{9c}(u) = 1$. By the Brauer table given above we have $\Theta_3(u^3) \sim \operatorname{diag}(1, \zeta_9^3, \zeta_9^6)$ and as φ_3 has only real values we get $\Theta_3(u) \sim \operatorname{diag}(1, \gamma, \delta)$ with $(\gamma, \delta) \in \{(\zeta_9, \zeta_9^8), (\zeta_9^2, \zeta_9^7), (\zeta_9^4, \zeta_9^5)\}$. Hence, with x, y, z as in the Brauer table,

$$x\varepsilon_{9a}(u) + y\varepsilon_{9b}(u) + z\varepsilon_{9c}(u) \in \{1 + \zeta_9 + \zeta_9^8, 1 + \zeta_9^2 + \zeta_9^7, 1 + \zeta_9^4 + \zeta_9^5\}.$$

Substituting x, y, and z and using ζ_9^2, ζ_9^3, ζ_9^4, ζ_9^5, ζ_9^6, ζ_9^7 as a basis of $\mathbb{Z}[\zeta_9]$ (this is a basis by [Neu92, Satz 10.2]) we obtain

$$(-\varepsilon_{9b}(u) + \varepsilon_{9c}(u), \varepsilon_{9a}(u) - \varepsilon_{9b}(u)) \in \{(-1, -1), (1, 0), (0, 1)\}.$$

Combining each possibility with $\varepsilon_{9a}(u) + \varepsilon_{9b}(u) + \varepsilon_{9c}(u) = 1$ this gives $(\varepsilon_{9a}(u), \varepsilon_{9b}(u), \varepsilon_{9c}(u)) \in \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$. So u is rationally conjugate to a group element.

Now let u be of order 10 and set $\zeta = \zeta_5$. If u is not rationally conjugate to a group element, so is u^3 and if u^2 is rationally conjugate to an element in $5a$, then u^6 is rationally conjugate to an element in $5b$. So we may assume that u^2 is conjugate to an element in

$5a$. We have

$$\varepsilon_{2a}(u) + \varepsilon_{5a}(u) + \varepsilon_{5b}(u) + \varepsilon_{10a}(u) + \varepsilon_{10b}(u) = 1.$$

By the Brauer table above we obtain $\Theta_3(u^5) \sim \mathrm{diag}(1, -1, -1)$ and $\Theta_3(u^6) \sim \mathrm{diag}(1, \zeta^2, \zeta^3)$. As φ_3 has only real values, we get $\Theta_3(u) \sim \mathrm{diag}(1, -\zeta^2, -\zeta^3)$. Thus

$$\begin{aligned} -\varepsilon_{2a}(u) \ + \ (-\zeta^2 - \zeta^3)\varepsilon_{5a}(u) \ + \ (-\zeta - \zeta^4)\varepsilon_{5b}(u) \\ + (-2\zeta - \zeta^2 - \zeta^3 - 2\zeta^4)\varepsilon_{10a}(u) \ + \ (-\zeta - 2\zeta^2 - 2\zeta^3 - \zeta^4)\varepsilon_{10b}(u) \ &= \ 1 - \zeta^2 - \zeta^3. \end{aligned}$$

Using $\zeta,\ \zeta^2,\ \zeta^3,\ \zeta^4$ as a basis of $\mathbb{Z}[\zeta]$ we obtain

$$\begin{aligned} \varepsilon_{2a}(u) - \varepsilon_{5b}(u) - 2\varepsilon_{10a}(u) - \varepsilon_{10b}(u) &= -1, \\ \varepsilon_{2a}(u) - \varepsilon_{5a}(u) - \varepsilon_{10a}(u) - 2\varepsilon_{10b}(u) &= -2. \end{aligned}$$

The same way we get $\Theta_5(u^5) \sim \mathrm{diag}(1, 1, 1, -1, -1)$, $\Theta_5(u^6) \sim \mathrm{diag}(1, \zeta, \zeta^2, \zeta^3, \zeta^4)$ and $\Theta_5(u) \sim \mathrm{diag}(X)$ with $X \in \{(1, -\zeta, \zeta^2, \zeta^3, -\zeta^4), (1, \zeta, -\zeta^2, -\zeta^3, \zeta^4)\}$. We have $\varphi_5(u) = \varepsilon_{2a}(u) - 2(\zeta + \zeta^4)\varepsilon_{10a}(u) - 2(\zeta^2 + \zeta^3)\varepsilon_{10b}(u)$. Hence

$$(-\varepsilon_{2a}(u) - 2\varepsilon_{10a}(u), -\varepsilon_{2a}(u) - 2\varepsilon_{10b}(u)) \in \{(-2, 0), (0, -2)\}.$$

Combining these equations with the equations obtained above we get $(\varepsilon_{2a}(u), \varepsilon_{5a}(u), \varepsilon_{5b}(u), \varepsilon_{10a}(u), \varepsilon_{10b}(u)) \in \{(0, 1, -1, 1, 0), (0, 0, 0, 0, 1)\}$. The possible partial augmentations $(\varepsilon_{2a}(u), \varepsilon_{5a}(u), \varepsilon_{5b}(u), \varepsilon_{10a}(u), \varepsilon_{10b}(u)) = (0, 1, -1, 1, 0)$ can not be eliminated using analogues computations with other characters. We will apply the observations of §1 here.

	1a	2a
φ_1	1	1
φ_{18}	18	-2

(a) Part of the Brauer table

	φ_1	φ_{18}
χ_1	1	·
χ_{18}	·	1
χ_{19}	1	1

(b) Part of the decomposition matrix

Table 3: Part of Brauer table and decomposition matrix of PSL(2, 19) for the prime 5

As D_{19} is a deleted permutation representation (i.e. the module corresponding to the representation is isomorphic to a permutation module factored by the trivial submodule) coming from the action of PSL(2, 19) on the projective line over $\mathbb{F}_{19}$, we may assume that D_{19} is a $\mathbb{Z}$-representation, so also a $\mathbb{Z}_5$-representation. By a theorem of Fong [Isa76, Cor. 10.13] we may assume that D_{18} is a K-representation, where K is an unramified extension of $\mathbb{Q}_5(\zeta+\zeta^4)$. Denote by R the ring of integers of K. As always denote by $\bar{\ }$the reduction modulo the maximal ideal of $\mathbb{Z}_5$ and of R. We may view $\bar{\mathbb{Z}}_5$ as a subfield of $\bar{R} =: k$.

Note that $\bar{L}_{19}$ contains $\bar{L}_{18}$ as submodule (multiplying a module by the augemntation ideal $I(kG)$ annihilates precisely the trivial kG-submodules). $\bar{L}_{19}/\bar{L}_{18}$ is a trivial kG-module, so also a trivial $k\langle\bar{u}\rangle$-module. By Proposition 1.3, slightly abusing the notation, as an $R\langle u\rangle$-lattice and as $\mathbb{Z}_5\langle u\rangle$-lattice we may write $L_{18} \cong L_{18}^1 \oplus L_{18}^{-1}$ and $L_{19} \cong L_{19}^1 \oplus L_{19}^{-1}$ resp. such that the composition factors of $\bar{L}_i^1$ are all trivial and the composition factors of $\bar{L}_i^{-1}$ are all non-trivial as $k\langle\bar{u}\rangle$-modules for $i \in \{18, 19\}$. As $\bar{L}_{19}/\bar{L}_{18}$ is a trivial module, we have $\bar{L}_{18}^{-1} \cong \bar{L}_{19}^{-1}$ (as $k\langle\bar{u}\rangle$-modules).

Since u^6 and u^5 are rationally conjugate to an element in 5b and 2a resp. we can compute the eigenvalues of $D_{19}(u^6)$ and $D_{19}(u^5)$ using the character table given above. Using the partial augmentations of u we then compute the eigenvalues of $D_{19}(u)$, which are not 5-th roots of unity, i.e. which contribute to L_{19}^{-1} by Proposition 1.3, to be $(-1, -\zeta, -\zeta^2, -\zeta^3, -\zeta^4, -1, -\zeta, -\zeta^2, -\zeta^3, -\zeta^4)$. By Proposition 1.4 this implies, slightly abusing the notation by making it intuitive: $L_{19}^{-1} \cong X$ with $X \in \{2(\mathbb{Z}_5)_{-1} \oplus 2I(\mathbb{Z}_5C_5)_{-1}$, $(\mathbb{Z}_5)_{-1} \oplus I(\mathbb{Z}_5C_5)_{-1} \oplus (\mathbb{Z}_5C_5)_{-1},\ 2(\mathbb{Z}_5C_5)_{-1}\}$. In any case $\bar{L}_{19}^{-1}$ has two indecomposable summands of k-dimension at least 4, as indecomposable summands of X stay indecomposable after reduction by Proposition 1.4.

On the other hand the eigenvalues of $D_{18}(u)$, which are not 5-th roots of unity are $(-1, -\zeta, -\zeta^2, -\zeta^3, -\zeta^4, -1, -\zeta, -\zeta^4, -\zeta, -\zeta^4)$. Note that the simple $R\langle u\rangle$-module S affording the eigenvalues $-\zeta^2$ and $-\zeta^3$ appears exactly once as a composition factor of L_{18}^{-1}. Let $L_{18}^{-1} \cong Y \oplus Z$ such that Y is indecomposable and S is a composition factor of Y. There are at most 2 different simple $K\langle u\rangle$-modules involved in Z, namely the one affording eigenvalues $-\zeta$ and $-\zeta^4$ and the one affording the eigenvalue -1. Hence by Remark 1.5 b) the maximal R-rank of an indecomposable summand of Z is 3. On the other hand up to isomorphism there are exactly 3 simple $K\langle u^2\rangle$-modules and so by Remark 1.5 c) the maximal R-rank of Y is 6. As the Krull-Schmidt-Azumaya Theorem holds, we obtain a contradiction to $\bar{L}_{18}^{-1} \cong \bar{L}_{19}^{-1}$ and the above paragraph.

Remark: The Schur index of every irreducible character of $\mathrm{PSL}(2,q)$ is actually 1 [Sha83], so in the above proof we may assume $K = \mathbb{Q}_5(\zeta_5 + \zeta_5^4)$ and avoid using the Theorem of Fong. However Fong's theorem could be really helpful, if this method is used for other groups, as demonstrated above.

Proof of ZC for $\mathrm{PSL}(2,23)$: By Proposition 2.1 only normalized units of order 4 and 12 have to be checked. We give the relevant part of the Brauer table for $p = 23$ in table 4.

	1a	2a	3a	4a	6a	12a	12b
φ_3	3	-1	·	1	2	$1+\zeta_{12}+\zeta_{12}^{-1}$	$1-\zeta_{12}-\zeta_{12}^{-1}$
φ_5	5	1	-1	-1	1	$2+\zeta_{12}+\zeta_{12}^{-1}$	$2-\zeta_{12}-\zeta_{12}^{-1}$
φ_7	7	-1	1	-1	-1	$2+\zeta_{12}+\zeta_{12}^{-1}$	$2-\zeta_{12}-\zeta_{12}^{-1}$
φ_{11}	11	-1	-1	1	-1	1	1

Table 4: Part of the Brauer table of $\mathrm{PSL}(2,23)$ for $p = 23$.

Let $u \in \mathrm{V}(\mathbb{Z}G)$ be a unit of order 4. By [Her07, Prop. 6.5] $\varepsilon_{2a}(u) = 0$ and so u is conjugate to a group element.

Let $u \in \mathrm{V}(\mathbb{Z}G)$ be of order 12 and $\zeta = \zeta_{12}$. We will use $\zeta, \zeta^{-1}, \zeta^4, \zeta^8$ as a $\mathbb{Z}$-basis of $\mathbb{Z}[\zeta]$. This is a basis since $\varphi(12) = 4$, where φ denotes Euler's totient function, and $1 = -\zeta^4 - \zeta^8$, $\zeta^2 = -\zeta^8, \zeta^3 = \zeta - \zeta^{-1}$, $\zeta^5 = -\zeta^{-1}$, $\zeta^6 = -1 = \zeta^4 + \zeta^8$, $\zeta^7 = -\zeta$, $\zeta^9 = -\zeta + \zeta^{-1}$ and $\zeta^{10} = -\zeta^4$. We have

$$\varepsilon_{2a}(u) + \varepsilon_{3a}(u) + \varepsilon_{4a}(u) + \varepsilon_{6a}(u) + \varepsilon_{12a}(u) + \varepsilon_{12b}(u) = 1. \tag{1}$$

By the Brauer table above $\Theta_3(u^6) \sim \mathrm{diag}(1,-1,-1)$, $\Theta_3(u^9) \sim \mathrm{diag}(1,\zeta^3,\zeta^9)$ and $\Theta_3(u^4) \sim \mathrm{diag}(1,\zeta^4,\zeta^8)$. Thus, as φ_3 has only real values, $\Theta_3(u) \sim \mathrm{diag}(X)$ with $X \in \{(1,\zeta^5,\zeta^7),(1,\zeta,\zeta^{11})\} = \{(1,-\zeta^{-1},-\zeta),(1,\zeta,\zeta^{-1})\}$. Hence, by the Brauer table given above, we obtain $-\varepsilon_{2a}(u) + \varepsilon_{4a}(u) + 2\varepsilon_{6a}(u) + (1+\zeta+\zeta^{-1})\varepsilon_{12a}(u) + (1-\zeta-\zeta^{-1})\varepsilon_{12b}(u) \in \{1+\zeta+\zeta^{-1}, 1-\zeta-\zeta^{-1}\}$. Using ζ, ζ^{-1}, ζ^4, ζ^8 as a basis of $\mathbb{Z}[\zeta]$ this gives

$$\varepsilon_{12a}(u) - \varepsilon_{12b}(u) = \pm 1, \tag{2}$$

$$-\varepsilon_{2a}(u) + \varepsilon_{4a}(u) + 2\varepsilon_{6a}(u) + \varepsilon_{12a}(u) + \varepsilon_{12b}(u) = 1. \tag{3}$$

Proceeding in the same way we get $\Theta_5(u^6) \sim \mathrm{diag}(1,1,1,-1,-1)$,
$\Theta_5(u^9) \sim \mathrm{diag}(1,-1,-1,\zeta^3,\zeta^9)$, $\Theta_5(u^4) \sim \mathrm{diag}(1,\zeta^4,\zeta^8,\zeta^4,\zeta^8)$ and $\Theta_5(u) \sim \mathrm{diag}(X)$ with

$X \in \{(1,\zeta^2,\zeta^{10},\zeta,\zeta^{11}),(1,\zeta^2,\zeta^{10},\zeta^5,\zeta^7)\}$. So, by the Brauer table and $\zeta^2+\zeta^{10}=1$, we get $\varepsilon_{2a}(u)-\varepsilon_{3a}(u)-\varepsilon_{4a}(u)+\varepsilon_{6a}(u)+(2+\zeta+\zeta^{-1})\varepsilon_{12a}(u)+(2-\zeta-\zeta^{-1})\varepsilon_{12b}(u) \in \{2+\zeta+\zeta^{-1}, 2-\zeta-\zeta^{-1}\}$. Comparing coefficients of ζ^4 gives

$$\varepsilon_{2a}(u)-\varepsilon_{3a}(u)-\varepsilon_{4a}(u)+\varepsilon_{6a}(u)+2\varepsilon_{12a}(u)+2\varepsilon_{12b}(u)=2. \tag{4}$$

Applying the same for φ_7 we obtain $\Theta_7(u^6) \sim \mathrm{diag}(1,1,1,-1,-1,-1,-1)$, $\Theta_7(u^9) \sim \mathrm{diag}(1,-1,-1,\zeta^3,\zeta^9,\zeta^3,\zeta^9)$, $\Theta_7(u^4) \sim \mathrm{diag}(1,\zeta^4,\zeta^8,1,\zeta^4,\zeta^8,1)$ and $\Theta_7(u) \sim \mathrm{diag}(X)$ with $X \in \{(1,-1,-1,\zeta,\zeta^{11},\zeta,\zeta^{11}),(1,-1,-1,\zeta,\zeta^{11},\zeta^5,\zeta^7),$ $(1,-1,-1,\zeta^5,\zeta^7,\zeta^5,\zeta^7),(1,\zeta^2,\zeta^{10},\zeta^3,\zeta^9,\zeta,\zeta^{11}),(1,\zeta^2,\zeta^{10},\zeta^3,\zeta^9,\zeta^5,\zeta^7)\}$. So, by the Brauer table, $\zeta^2+\zeta^{10}=1$, and $\zeta^3+\zeta^9=0$, we get $\varepsilon_{2a}(u)+\varepsilon_{3a}(u)-\varepsilon_{4a}(u)-\varepsilon_{6a}(u)+(2+\zeta+\zeta^{-1})\varepsilon_{12a}(u)+(2-\zeta-\zeta^{-1})\varepsilon_{12b}(u) \in \{-1+2\zeta+2\zeta^{-1},\ -1,\ -1-2\zeta-2\zeta^{-1},\ 2+\zeta+\zeta^{-1},\ 2-\zeta-\zeta^{-1}\}$. As the first three possibilities would give $\varepsilon_{12a}(u)-\ varepsilon_{12b}(u) \in \{-2,0,2\}$, contradicting (2), only the last two remain and give

$$-\varepsilon_{2a}(u)+\varepsilon_{3a}(u)-\varepsilon_{4a}(u)-\varepsilon_{6a}(u)+2\varepsilon_{12a}(u)+2\varepsilon_{12b}(u)=2. \tag{5}$$

The same way we get $\Theta_{11}(u^6) \sim \mathrm{diag}(1,1,1,1,1,-1,-1,-1,-1,-1,-1)$, $\Theta_{11}(u^9) \sim \mathrm{diag}(1,1,1,-1,-1,\zeta^3,\zeta^9,\zeta^3,\zeta^9,\zeta^3,\zeta^9)$, $\Theta_{11}(u^4) \sim \mathrm{diag}(1,\zeta^4,\zeta^8,1,\zeta^4,\zeta^8,1,\zeta^4,\zeta^8,\zeta^4,\zeta^8)$, and $\Theta_{11}(u) \sim \mathrm{diag}(1,\zeta,\zeta^2,\zeta^3,\zeta^4,\zeta^5,\zeta^7,\zeta^8,\zeta^9,\zeta^{10},\zeta^{11})$ (note that $\varphi_{11}(u)$ must not only be real valued, but even rational as φ_{11} has only rational values). Thus $-\varepsilon_{2a}(u)-\varepsilon_{3a}(u)+\varepsilon_{4a}(u)-\varepsilon_{6a}(u)+\varepsilon_{12a}(u)+\varepsilon_{12b}(u)=1$ giving

$$-\varepsilon_{2a}(u)-\varepsilon_{3a}(u)+\varepsilon_{4a}(u)-\varepsilon_{6a}(u)+\varepsilon_{12a}(u)+\varepsilon_{12b}(u)=1. \tag{6}$$

Now subtracting (1) from (6) gives $\varepsilon_{2a}(u)+\varepsilon_{3a}(u)+\varepsilon_{6a}(u)=0$ while subtracting (4) from (5) gives $\varepsilon_{2a}(u)-\varepsilon_{3a}(u)+\varepsilon_{6a}(u)=0$. Thus $\varepsilon_{3a}(u)=0$. Then subtracting (1) from (3) gives $-2\varepsilon_{2a}(u)+\varepsilon_{6a}(u)=0$, so $\varepsilon_{2a}(u)=\varepsilon_{6a}(u)=0$. Now multiplying (1) by 2 and subtracting it from (4) gives $\varepsilon_{4a}(u)=0$. Using (1) and (2) this leaves only the trivial possibilities $(\varepsilon_{2a}(u),\varepsilon_{3a}(u),\varepsilon_{4a}(u),\varepsilon_{6a}(u),\varepsilon_{12a}(u),\varepsilon_{12b}(u)) \in \{(0,0,0,0,1,0),(0,0,0,0,0,1)\}$. Thus u is rationally conjugate to a group element and Theorem 1 is proved. □

2.2 Proof of Theorem 2

By [KK12] if a unit u of order 6 exists in $V(\mathbb{Z}M_{10})$ or in $V(\mathbb{Z}\,\mathrm{PGL}(2,9))$, then all partial augmentations of u vanish except at the elements of order 2 and 3 in A_6 (note that A_6 is a normal subgroup of index 2 in M_{10} and in $\mathrm{PGL}(2,9)$) and then $(\varepsilon_{2a}(u), \varepsilon_{3a}(u)) = (-2, 3)$. As both groups M_{10} and $\mathrm{PGL}(2,9)$ are subgroups of $\mathrm{Aut}(A_6)$ and the conjugacy classes $2a$ and $3a$ are the same in all these groups, we will handle both cases at once showing, that there is no unit of order 6 in $V(\mathbb{Z}\mathrm{Aut}(A_6))$ having partial augmentations $(\varepsilon_{2a}(u), \varepsilon_{3a}(u)) = (-2, 3)$.

The relevant parts of the character tables are given in table 5, the corresponding decomposition matrix in table 6.

	1a	2a	3a
χ_{1a}	1	1	1
χ_{1b}	1	1	1
χ_{1c}	1	1	1
χ_{1d}	1	1	1
χ_{10a}	10	2	1
χ_{16a}	16	·	−2
χ_{16b}	16	·	−2
χ_{20}	20	−4	2

(a) Used part of the ordinary character table

	1a	2a
φ_{1a}	1	1
φ_{1b}	1	1
φ_{1c}	1	1
φ_{1d}	1	1
φ_{6a}	6	−2
φ_{6b}	6	−2
φ_{8}	8	·

(b) Used part of the Brauer table for $p = 3$:

Table 5: Parts of ordinary character table and Brauer table for the prime 3 for the group $\mathrm{Aut}(A_6)$

	φ_{1a}	φ_{1b}	φ_{1c}	φ_{1d}	φ_{6a}	φ_{6b}	φ_{8}
χ_{1a}	1	·	·	·	·	·	·
χ_{1b}	·	1	·	·	·	·	·
χ_{1c}	·	·	1	·	·	·	·
χ_{1d}	·	·	·	1	·	·	·
χ_{10a}	1	1	·	·	·	·	1
χ_{16a}	1	·	1	·	1	·	1
χ_{16b}	·	1	·	1	·	1	1
χ_{20}	·	·	·	·	1	1	1

Table 6: Part of the decomposition matrix of $\mathrm{Aut}(A_6)$ for the prime 3

Set $G = \mathrm{Aut}(A_6)$ and let u be a unit of order 6 in $V(\mathbb{Z}G)$ such that $(\varepsilon_{2a}(u), \varepsilon_{3a}(u)) =$

$(-2,3)$ and $\varepsilon_g(u)=0$ for all other conjugacy classes ($2a$ is the $\mathrm{Aut}(A_6)$-conjugacy class of involutions of length 45 and $3a$ is the unique $\mathrm{Aut}(A_6)$-conjugacy class of elements of order 3). Denote by ζ a complex primitive 3rd root of unity. Using the HeLP-method and the fact that each χ_i is real valued, we obtain

$$D_{10a}(u) \sim \mathrm{diag}(1,\zeta,\zeta^2,1,\zeta,\zeta^2,-1,-\zeta,-\zeta^2,-1)$$
$$D_{16a}(u) \sim D_{16b}(u) \sim \mathrm{diag}(\zeta,\zeta^2,\zeta,\zeta^2,\zeta,\zeta^2,\zeta,\zeta^2,-1,-\zeta,-\zeta^2,-1,-\zeta,-\zeta^2,-1,-1)$$
$$D_{20}(u) \sim \mathrm{diag}(1,1,1,1,1,1,1,1,-\zeta,-\zeta^2,-\zeta,-\zeta^2,-\zeta,-\zeta^2,-\zeta,-\zeta^2,-\zeta,-\zeta^2,-\zeta,-\zeta^2)$$

These can be computed in the way demostrated above. We give one example: As u^4 is rationally conjugate to an element in 3a and u^3 is rationally conjugate to an element in 2a, we have $\chi_{10}(u^4)=\chi_{10}(3a)=1$ and $\chi_{10}(u^3)=\chi_{10}(2a)=2$. This gives $D_{10}(u^4)\sim \mathrm{diag}(1,\zeta,\zeta^2,1,\zeta,\zeta^2,1,\zeta,\zeta^2,1)$ and $D_{10}(u^3)\sim\mathrm{diag}(1,1,1,1,1,1,-1,-1,-1,-1)$. Now $\chi_{10}(u)=\varepsilon_{2a}(u)\chi_{10}(2a)+\varepsilon_{3a}(u)\chi_{10}(3a)=-1$ and as the eigenvalues of $D_{10}(u)$ are products of the eigenvalues of $D_{10}(u^4)$ and $D_{10}(u^3)$ this gives
$D_{10}(u)\sim\mathrm{diag}(1,\zeta,\zeta^2,1,\zeta,\zeta^2,-1,-\zeta,-\zeta^2,-1)$.
As all the character values of all ordinary characters of G are integers on all conjugacy classes of G, we may assume by a theorem of Fong [Isa76, Cor. 10.13] that all ordinary representations mentioned above are K-representations, where K is an unramified extension of $\mathbb{Q}_3$. So if R is the ring of integers of K we may assume that they are even R-representations. Let P be the maximal ideal of R, set $k=R/P$ and let $^-$ denote the reduction modulo P. Denote by L_* an RG-lattice affording the representation D_*. Denote by k, $I(kC_3)$, and kC_3 the indecomposable kC_6 modules of k-dimension 1, 2, and 3 resp. having trivial composition factors and by $(k)_-$, $I(kC_3)_-$, and $(kC_3)_-$ the indecomposable kC_6-modules of k-dimension 1, 2, and 3 resp. having non-trivial composition factors (see Propositions 1.2 and 1.4). We will write M_* for a simple kG-module having character φ_*. Regarded as $k\langle\bar{u}\rangle$-modules using Propositions 1.2 and 1.3 we will write $\bar{L}_*\cong\bar{L}_*^1\oplus\bar{L}_*^{-1}$ and $M_*\cong M_*^1\oplus M_*^{-1}$, where all the composition factors of M_*^1 and $\bar{L}_*^1$ are trivial and all the composition factors of M_*^{-1} and $\bar{L}_*^{-1}$ are non-trivial. As u^3 is rationally conjugate to an element in $2a$ it is also 3-adically conjugate to this element by [Her06, Lemma 2.9]. Thus the k-dimensions of M_*^1 and M_*^{-1} can be deduced from the Brauer table above. The k-dimensions of $\bar{L}_*^1$ and $\bar{L}_*^{-1}$ can be deduced from the eigenvalues given above using Proposition 1.3. We give the dimensions in table 7.

$k\langle\bar{u}\rangle$-module M	k-dimension of M^1	k-dimension of M^{-1}
$\bar{L}_{10a}$	6	4
$\bar{L}_{16*}$	8	8
$\bar{L}_{20}$	8	12
M_{1*}	1	0
M_{6*}	2	4
M_8	4	4

Where $*$ takes all possible values in $\{a, b, c, d\}$.

Table 7: Dimensions of M^1 and M^{-1} for certain $k\langle\bar{u}\rangle$-modules

The Krull-Schmidt-Azumaya Theorem will be used without further mentioning. We will use decomposition series of $\bar{L}_*$ as kG-module, which we obtained using the GAP package MeatAxe [GAP12][2] , as shown in table 8.

kG-module M	Socle of M	Head of M
$\bar{L}_{10a}$	M_{1i}	M_{1j}
$\bar{L}_{16a}$	$M_{6a} \oplus M_{1a} \oplus M_{1c}$	M_8
$\bar{L}_{16b}$	$M_{6b} \oplus M_{1b} \oplus M_{1d}$	M_8
$\bar{L}_{20}$	$M_{6a} \oplus M_{6b}$	M_8

Where (i, j) takes a value in $\{(a, b), (b, a)\}$.

Table 8: Decomposition factors of certain reduced $\mathbb{Z}\mathrm{Aut}(A_6)$-lattices

From now on all modules will be $k\langle\bar{u}\rangle$-modules. With the eigenvalues of $D_{20}(u)$ as above using Propositions 1.3 and 1.4 we get $\bar{L}_{20}^{-1} \cong 6I(kC_3)_-$. As all M_{1*} are trivial $k\langle\bar{u}\rangle$-modules by the Brauer table given above, using the eigenvalues of $D_{10a}(u)$ and Proposition 1.4, we obtain $M_8^{-1} \cong \bar{L}_{10}^{-1} \cong X$ with $X \in \{(k)_- \oplus (kC_3)_-, 2(k)_- \oplus I(kC_3)_-\}$. But as $M_8^{-1} \cong \bar{L}_{20}^{-1}/(M_{6a}^{-1} \oplus M_{6b}^{-1})$, i.e. M_8^{-1} is also a quotient of $\bar{L}_{20}^{-1} \cong 6I(kC_3)_-$, we get $M_8^{-1} \cong 2(k)_- \oplus I(kC_3)_-$. So $6I(kC_3)_-/(M_{6a}^{-1} \oplus M_{6b}^{-1}) \cong \bar{L}_{20}^{-1}/(M_{6a}^{-1} \oplus M_{6b}^{-1}) \cong M_8^{-1} \cong 2(k)_- \oplus I(kC_3)_-$ and this implies $M_{6a}^{-1} \oplus M_{6b}^{-1} \cong 2(k)_- \oplus 3I(kC_3)_-$. As $\dim_k(M_{6a}^{-1}) = \dim_k(M_{6b}^{-1}) = 4$, this gives either $M_{6a}^{-1} \cong 2(k)_- \oplus I(kC_3)_-$ and $M_{6b}^{-1} \cong 2I(kC_3)_-$ or $M_{6a}^{-1} \cong 2I(kC_3)_-$ and $M_{6b}^{-1} \cong 2(k)_- \oplus I(kC3)_-$. Consider the first possibility, the other one follows in an analogues way interchanging a and b. Now by the eigenvalues of $D_{16b}(u)$ computed above

[2]The representations of irreducible modules are available in `GAP` by the command `IrreducibleRepresentationsDixon` or `AffordingIrreducibleRepresentation` once the character is given.

we get $\bar{L}_{16b}^{-1} \cong Y$ with $Y \in \{2(k)_- \oplus 2(kC_3)_-,\ 3(k)_- \oplus I(kC_3)_- \oplus (kC_3)_-,\ 4(k)_- \oplus 2I(kC_3)_-\}$ by Proposition 1.3 and 1.4. In every case, as all M_{1*} are trivial modules, we get $M_8^{-1} \cong \bar{L}_{16b}^{-1}/M_{6b}^{-1} \cong \bar{L}_{16b}^{-1}/2I(kC_3)_- \cong 4(k)_-$. Contradicting $M_8^{-1} \cong 2(k)_- \oplus I(kC_3)_-$. □

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Andreas Bächle, Vakgroep Wiskunde, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium. *ABachle@vub.ac.be*

Leo Margolis, Fachbereich Mathematik, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany. *leo.margolis@mathematik.uni-stuttgart.de*

Andreas Bächle
Vakgroep Wiskunde, Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium
E-Mail: ABachle@vub.ac.be
WWW: http://homepages.vub.ac.be/∼abachle/

Leo Margolis
Fachbereich Mathematik, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany
E-Mail: leo.margolis@mathematik.uni-stuttgart.de
WWW: http://www.igt.uni-stuttgart.de/LstDiffgeo/Margolis/

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