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Hausdorff Dimension of Rings

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HAUSDORFF DIMENSION OF RINGS

Dietmar Kahnert

Dedicated to Professor Dr. Bodo Volkmann on his 85. birthday

Abstract

In context with the problem of Volkmann whether a subfield K of $\mathbb{R}$ exists with Hausdorff dimension $\dim K \in (0,1)$, Falconer has proven that there is no subring S with $1/2 < \dim S < 1$ which is an analytic set. We prove that $S = \mathbb{R}$ for every such subring S with $\dim S > 0$.

1 A problem of Volkmann

A function $h : [0,\infty) \to [0,\infty]$ is called Hausdorff function if the following is valid: $h(0) = 0$, $h(t) > 0$ if $t > 0$, $h(a) \leq h(b)$ if $a \leq b$ and $\lim_{t\to 0} h(t) = 0$.

Let H be the set of all Hausdorff functions. For each $h \in H$ and every subset A of $\mathbb{R}^n$ is the outer measure

$$L_h(A) = \lim_{q\to 0} \inf \left\{ \sum_{i=1}^{\infty} h(d(A_i) : A = \bigcup_{i=1}^{\infty} A_i, d(A_i) \leq q \text{ for all } i \in \mathbb{N} \right\}$$

defined. Let $d(A_i)$ be the diameter of A_i. Souslin sets (also called “analytic sets”), especially Borel sets, are L_h-measurable. If $h(t) = t^\alpha$ $(\alpha > 0)$, $L_\alpha = L_h$ is the α-dimensional Hausdorff measure and

$$\dim A = \sup\{\alpha : L_\alpha(A) > 0\}$$

the Hausdorff dimension of A.

The field problem of Volkmann [18]

Is there a subfield K of the field $\mathbb{R}$ of real numbers with $0 < \dim K < 1$?

The problem still remains open.

Result of Falconer ([5] and [7])

No Souslin subring S of $\mathbb{R}$ exists with $1/2 < \dim S < 1$.

The result of Falconer emerges from his theorems regarding the projections of subsets E of $\mathbb{R}^2$ onto $\mathbb{R}$ and over distance sets $D(E) = \{|x-y| : x, y \in E\}$. They were won with the help of Fourier transformations. The following generalization should be treated here (Theorem 2):

For each Souslin subring S of $\mathbb{R}$ with $\dim S > 0$ is $S = \mathbb{R}$.

2 Special subfields of $\mathbb{R}$

2.1 Small subfields

An uncountable F_σ-subfield K of $\mathbb{R}$ with $L_1(A) = 0$ is constructed in a paper of Souslin [17]. In measure-theoretical view one can win **small** subfields K of $\mathbb{R}$ with help of the metric dimension of Wegmann [19]. Wegmann defines for subsets A of $\mathbb{R}^n$ and $q > 0$

$$N(A,q) = \min\{k: \text{ There are sets } A_1, \dots, A_k \text{ such that } \bigcup_{i=1}^{k} A_i = A;\ d(A_i) \le q \text{ if } 1 \le i \le k\}$$

and

$$m\text{-}\dim A = \sup\{s: \text{ If } \bigcup_{i=1}^{\infty} A_i = A, \text{ then exists } i \in \mathbb{N} \text{ with } \limsup_{q\to 0} N(A_i,q)q^s > 0\}.$$

In the book of Mattila [14] m-dim $= \overline{\dim}_p$ is called **upper packing dimension**. Clearly dim $\le m$-dim.

If A is a subset of $\mathbb{R}$ and $K(A)$ the smallest subfield of $\mathbb{R}$ containing A:
m-dim $A = 0 \to m$-dim $K(A) = 0$ (Kahnert [9]).

With the method used in [9] we can prove: If $g, h \in H$ and $\lim_{q\to 0} h(t)/g(t)^n = 0$ for all $n \in \mathbb{N}$, then $\lim_{q\to 0} N(A,q)g(q) = 0 \to L_h(K(A)) = 0$ for every compact subset A of $\mathbb{R}$.

An uncountable subset A of $\mathbb{R}$ is called **Lusin set**, if every uncountable subset of A is of second category. For Lusin sets A is $L_h(A) = 0$ for each $h \in H$.

With the help of the continuum hypothesis it is possible to get subfields of $\mathbb{R}$ which are Lusin sets (Erdös [4]).

2.2 Big subfields

According to an idea of Zygmund, in the paper of Souslin [17], the existence of a non-L_1-measureable subfield K of $\mathbb{R}(L_1(K) > 0, \dim K = 1)$ can be proven with the help of the axiom of choice.

An uncountable subset A of $\mathbb{R}$ is called **Sierpinski set** (dual to Lusin set), if every uncountable subset of A is of positive outer L_1-measure. With the help of the continuum hypothesis Erdös and Volkmann [3] proved the existence of fields which are Sierpinski sets.

In the latter mentioned paper Erdös und Volkmann constructed for each $\alpha \in (0,1)$ additive F_σ-subgroups $G(\alpha)$ of $\mathbb{R}$ with $\dim G(\alpha) = \alpha$ $(= m\text{-}\dim G(a))$. This result led to the supposition that a corresponding statement could be true for subfields of $\mathbb{R}$.

3 Subfields K of the complex numbers $\mathbb{C}$ of finite degree

A field E containing a field F can be regarded as an F-vector space. We write $E : F$ for the dimension. We refer in the following to the book of Hornfeck [8].

3.1 $\mathbb{C}$ is normal over K if $\mathbb{C} : K < \infty$

Let $G(\mathbb{C} : K)$ be the group of automorphism φ of $\mathbb{C}$ with $\varphi(x) = x$ for all $x \in K$. The field $\mathbb{C}$ is called **normal** over K if K is the fixed field of $G(\mathbb{C} : K)$ (other authors name in this case $\mathbb{C}$

Galois over K). There is $\alpha \in \mathbb{C}$ with $\mathbb{C} = K(\alpha)$ (Theorem 3a, 61.2). The minimal polynomial $f(x) \in K[x]$ of α splits in $\mathbb{C}[x]$:

$$f(x) = (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_n), \quad (\alpha_1 = \alpha).$$

Therefore is $K(\alpha_1, \alpha_2, \ldots, \alpha_n) = K(\alpha) = \mathbb{C}$ a splitting field of $f(x)$ (Lemma in 58.1). $\mathbb{C}$ is normal over K (Theorem 7, 65.2) and $|G(\mathbb{C} : K)| = \mathbb{C} : K$.

The field $\mathbb{C}$ has two continuous automorphisms φ_1 and φ_2 with $\varphi_1(x) = x$ and $\varphi_2(x) = \overline{x}$ for all $x \in \mathbb{C}$. The field $\mathbb{R}$ has only one automorphism.

3.2 Continuous additive functions

We shall prove Theorem 1 using the following special-case of a result of Ostrowski [15] and give the proof due to Kestelman [11].

Ostrowski's theorem If $f : \mathbb{R}^n \to \mathbb{R}^n$ is additive ($f(x + y) = f(x) + f(y)$) and bounded on a L_n-measurable set A with $L_n(A) > 0$, then f is continuous.

Proof: Let $M = \sup\{|f(x)| : x \in A\}$. After a known result of Steinhaus, the set $A - A$ contains a ball around the origin with radius r. Every $x \in \mathbb{R}^n$ with $|x| < r$ can be written as $x = a - b$ $(a, b \in A)$ and therefore

$$|f(x)| = |f(a) - f(b)| \leq 2M.$$

For $n \in \mathbb{N}$ and $|x| < r/n$ is $|nx| < r$, $|f(nx)| = n\,|f(x)| \leq 2M$ and $|f(x)| \leq 2M/n$. Therefore f is continuous in the origin and consequently everywhere continuous. □

3.3 Souslin subfields K of $\mathbb{C}$ with $\mathbb{C} : K < \infty$

The properties of analytic sets mentioned here are treated for example in the book of Parthasarathy [16].

Theorem 1 *Souslin subfields K of $\mathbb{C}$ with $\mathbb{C} : K < \infty$ are only $K = \mathbb{R}$ and $K = \mathbb{C}$.*

Proof: Let $b_1, \ldots, b_n$ be a basis of $\mathbb{C}$ over K:

$$\mathbb{C} = b_1 K + b_2 K + \ldots + b_n K.$$

For $r > 0$ we define

$$\begin{aligned} A(r) &= \{b_1 z_1 + b_2 z_2 + \ldots + b_n z_n : (z_1, \ldots, z_n) \in K^n, |z_1| + \ldots + |z_n| \leq r\}, \\ B(r) &= \{(z_1, z_2, \ldots, z_n) \in \mathbb{C}^n : |z_1| + \ldots + |z_n| \leq r\} \text{ and} \\ C(r) &= B(r) \cap K^n. \end{aligned}$$

Then $B(r)$ is compact, K^n and $C(r)$ are Souslin sets in $\mathbb{C}^n$.

We show: $A(r)$ is analytic.

For $z = (z_1, \ldots, z_n)$ let

$$\begin{aligned} f(z) &= b_1 z_1 + \ldots + b_n z_n \qquad (z \in \mathbb{C}^n) \text{ and} \\ g(z) &= f(z) \qquad (z \in C(r)). \end{aligned}$$

Since f is continuous it follows that for every Borel set D in $\mathbb{C}$

$$f^{-1}(D) \text{ is a Borel set in } \mathbb{C}^n \text{ and}$$
$$g^{-1}(D) = f^1(D) \cap C(r) \text{ is a Borel set in } C(r).$$

Therefore g is Borel measurable and $A(r) = g(C(r))$ analytic.

For $\varphi \in G(\mathbb{C} : K)$ ($\mathbb{C}$ is normal over K) and $M = \max\{|b|, \ldots, b|\}$ is

$$|\varphi(z)| \leq Mr \text{ if } z \in A(r).$$

There is r with $L_2(A(r) > 0$. By Ostrowski's result φ is continuous, therefore $G(\mathbb{C} : K) = \{\varphi_1\}$ and $K = \mathbb{C}$, or $G(\mathbb{C} : K) = \{\varphi_1, \varphi_2\}$ and $K = \mathbb{R}$. □

The analytic property of K is not required with the following result.

Artin's Theorem [1] For every subfield K of $\mathbb{C}$ with $1 < \mathbb{C} : K < \infty$ is $\mathbb{C} : K = 2$. (Especially there exists no subfield K of $\mathbb{R}$ with $1 < \mathbb{R} : K < \infty$.)

The following special-case, that can be used in the Section 5, can easily be proven. There is no subfield K of $\mathbb{R}$ with $\mathbb{R} : K = 2$.

Suppose that K is a subfield of $\mathbb{R}$ and $\mathbb{R} : K = 2$. Then there is a real number α with $\mathbb{R} = K(\alpha)$. Let $f(x)$ be the minimal polynomial of α and

$$f(x) = (x - \alpha_1)(x - \alpha_2), \quad (\alpha_1 = \alpha).$$

Then α_2 must be a real number.

Thus is $K(\alpha_1, \alpha_2) = K(\alpha) = \mathbb{R}$, $K(\alpha_1, \alpha_2)$ a splitting field, $\mathbb{R}$ normal over K and $|G(\mathbb{R} : K)| = 2$. But $\mathbb{R}$ has only one automorphism.

4 The Main Result

Let A be a non-empty subset of $\mathbb{R}$ and $\mathbb{R}(A)$ the subring of $\mathbb{R}$ generated by A.

Theorem 2 *For every closed subset A of $\mathbb{R}$ with $\dim A > 0$ is $R(A) = \mathbb{R}$.*

By results of Besicovitch and Davies [2] any Souslin subset A of $\mathbb{R}^n$ with $L_\alpha(A) > 0$ contains a closed subset B with $0 < L_\alpha(B) < \infty$. For every Souslin subring S of $\mathbb{R}$ with $\dim S > 0$ is therefore $S = \mathbb{R}$.

We use the following theorems of Marstrand to prove $\mathbb{R} : K(A) < \infty$ for every set A of Theorem 2. For subsets E of $\mathbb{R}^2$ and $t \in \mathbb{R}$ be

$$E(t) = \{x + ty : (x, y) \in E\}.$$

Projection theorem (Marstrand [12]) Let E be a Souslin subset of $\mathbb{R}^2$ with $\dim E = \alpha$:

a) $\alpha \leq 1 : \dim E(t) = \alpha$ for almost all $t \in \mathbb{R}$,

b) $\alpha > 1 : L_1(E(t)) > 0$ for almost all $t \in \mathbb{R}$.

A potential theoretic proof was given by Kaufmann [10]. Generalizations can be found in the books of Falconer [6] and Mattila [14].

Product theorem (Marstrand [13]) For any subsets A und B of $\mathbb{R}^n$

$$\dim A \times B \geq \dim A + \dim B.$$

A generalization of the product formula for general metric spaces was proven by Wegmann [19].

Proof of Theorem 2.

1. With possibly multiple applications of the theorems of Marstrand one proves the following assertion:

   There are real numbers $b_1, \ldots, b_n$ with $L_1(b_1 A + \ldots + b_n A) > 0$.

   Let $b_1 = 1$ and $A_1 = A$. In the case $L_1(A) > 0$ there is nothing to prove. May $A_k = b_1 A + \ldots + b_k A$ be defined and $L_1(A_1) = \ldots = L_1(A_k) = 0$.

   In the case $\dim A_k \times A > 1$ exists by the projection theorem a real number b_{k+1} with $L_1(A_k + b_{k+1} A) > 0$ and the assertion is verified.

   In the case $\dim A_k \times A \leq 1$ there exists by the projection theorem a real number b_{k+1} with $\dim A_k + b_{k+1} A = \dim A_k \times A$. For $A_{k+1} = A_k + b_{k+1} A$ is by the product formula

   $$\dim A_{k+1} \geq \dim A_k + \dim A \geq (k+1) \dim A.$$

   After finite steps, one arrives at the assertion.

2. If U is the additive subgroup of $\mathbb{R}$ generated by A, then

   $$G = b_1 U + \ldots + b_n U$$

   is a group, by the theorem of Steinhaus a neighborhood of 0 and therefore $G = \mathbb{R}$. For $S = R(A)$ and for the F_σ-field

   $$K = \{s/t : s, t \in S, t \neq 0\} = K(A) \text{ is}$$
   $$b_1 K + \ldots + b_n K = \mathbb{R}, \mathbb{R} : K \leq n$$

   and therefore $K = \mathbb{R}$ (Artin's theorem, Theorem 1).

   Let $b_1 = s_1/t_1, \ldots, b_n = s_n/t_n (s_i, t_i \in S; t_1 t_2 \cdots t_n \neq 0)$.

   Multiplying $b_1 S + \ldots + b_n S = \mathbb{R}$ with $t_1 t_2 \cdots t$ we get

   $$\mathbb{R} = d_1 S + \ldots + d_n S = S (d_1, \ldots, d_n \in S), \mathbb{R} = S.$$

□

5 A special case of Theorem 2

For every closed subset A of $\mathbb{R}$ with $\dim A > 1/2$ is $R(A) = \mathbb{R}$.

Proof: It is $\dim A \times A \geq 2 \dim A > 1$. By the projection theorem (part b) there exists a real number t with

$$L_1(A + tA) > 0.$$

Let S be the F_σ-ring $R(A)$. Then $L_1(S + tS) > 0$.

By the theorem of Steinhaus is the additive group

$$S + tS = (S + tS) - (S + tS)$$

neighborhood of 0 and therefore $S + tS = \mathbb{R}$.
For the field $K = K(A) = \{a/b : a,\, b \in S,\, b \neq 0\}$ is

$$K + tK = \mathbb{R}, \quad \mathbb{R} : K \leq 2$$

and therefore $K = \mathbb{R}$ ($\mathbb{R} : K = 2$ is not possible).
Be $t = a/b (a, b \in S;\, b \neq 0)$. Then $bS + aS = \mathbb{R} = S$. □

6 Problems

- Is there a Souslin subfield K of $\mathbb{C}$ with

$$0 < \dim K < 2 \text{ and } \dim K \neq 1?$$

- Is there a subfield K of $\mathbb{R}$ with $0 < m\text{-}\dim K < 1$?
- Which possibilities are there for a subfield $K \neq \mathbb{R}$ of $\mathbb{C}$ with $\mathbb{C} : K = 2$ concerning $\dim K, m\text{-}\dim K$ and the Baire category of K?

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