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Fachbereich Mathematik

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Preprint 2014/017

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ISSN **1613-8309**

A Sylow theorem for the integral group ring of $\mathrm{PSL}(2,q)$

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August 26, 2014

Abstract: For $G = \mathrm{PSL}(2, p^f)$ denote by $\mathbb{Z}G$ the integral group ring over G and by $V(\mathbb{Z}G)$ the group of units of augmentation 1 in $\mathbb{Z}G$. Let r be a prime different from p. Using the so called HeLP-method we prove that units of r-power order in $V(\mathbb{Z}G)$ are rationally conjugate to elements of G. As a consequence we prove that subgroups of prime power order in $V(\mathbb{Z}G)$ are rationally conjugate to subgroups of G, if $p = 2$ or $f \leq 2$.

Let G be a finite group and $\mathbb{Z}G$ the integral group ring over G. Denote by $V(\mathbb{Z}G)$ the group of units of augmentation 1 in $\mathbb{Z}G$. We say that a finite subgroup U of $V(\mathbb{Z}G)$ is rationally conjugate to a subgroup W of G, if there exists a unit $x \in \mathbb{Q}G$ such that $x^{-1}Ux = W$. The question if some, or even all, finite subgroups of $V(\mathbb{Z}G)$ are rationally conjugate to subgroups of G was proposed by H. J. Zassenhaus in the '60s and published in [Zas74]. This so called Zassenhaus Conjectures motivated a lot of research. E.g. A. Weiss proved the strongest version, that all finite subgroups of $V(\mathbb{Z}G)$ are rationally conjugate to subgroups of G, provided G is nilpotent [Wei88] [Wei91]. K. W. Roggenkamp and L. L. Scott obtained a counterexample [Rog91] to this strong conjecture. The version, which asks whether all finite cyclic subgroups of $V(\mathbb{Z}G)$ are rationally conjugate to subgroups of G, the so called First Zassenhaus Conjecture, is however still open, see e.g. [Her08a], [CMdR13]. Though mostly solvable groups were considered when studying such questions, there are some results available for non-solvable series of groups. E.g. a work on the symmetric groups [Pet76] or for Lie-groups of small rank [Ble99]. The groups $\mathrm{PSL}(2, q)$, which are also the object of study in this paper, found also some special attention in [Wag95], [Her07], [HHK09] or in [BK11]. In this paper we

will limit our attention to finite p-subgroups of $V(\mathbb{Z}G)$.
One could ask, what a Sylow-like theorem could mean for $V(\mathbb{Z}G)$. One variation, lets say a **weak Sylow theorem**, would be that every finite p-subgroup of $V(\mathbb{Z}G)$ is isomorphic to some subgroup of G. A stronger result, say a **strong Sylow theorem**, would be, if every finite p-subgroup of $V(\mathbb{Z}G)$ is even rationally conjugate to a subgroup of G. First Sylow-like results for integral group rings were obtained in [KR93]. Later M. A. Dokuchaev and S. O. Juriaans proved a strong Sylow theorem for special classes of solvable groups [DJ96] and M. Hertweck, C. Höfert and W. Kimmerle proved a weak Sylow theorem for $\mathrm{PSL}(2,p^f)$, where $p = 2$ or $f \leq 2$. The results of this article are as follows:

Proposition 1: Let $G = \mathrm{PSL}(2,p^f)$, let r be a prime different from p and let u be a torsion unit in $V(\mathbb{Z}G)$ of r-power order. Then u is rationally conjugate to a group element.

Theorem 2: Let $G = \mathrm{PSL}(2,p^f)$ such that $f \leq 2$ or $p = 2$. Then a strong Sylow theorem holds in $V(\mathbb{Z}G)$.

1 HeLP-method and known results

Let G be a finite group. A very useful notion to study rational conjugacy of torsion units are partial augmentations: Let $u = \sum_{g \in G} a_g g \in \mathbb{Z}G$ and x^G be the conjugacy class of the element $x \in G$ in G. Then $\varepsilon_x(u) = \sum_{g \in x^G} a_g$ is called the **partial augmentation** of u at x. This relates to rational conjugacy via:

Lemma 1.1 ([MRSW87, Th. 2.5]). *Let $u \in \mathrm{V}(\mathbb{Z}G)$ be a torsion unit. Then u is rationally conjugate to a group element if and only if $\varepsilon_x(u^k) \geq 0$ for all $x \in G$ and all powers u^k of u.*

It is well known that if $u \neq 1$ is a torsion unit in $\mathrm{V}(\mathbb{Z}G)$, then $\varepsilon_1(u) = 0$ by the so called Berman-Higman Theorem [Seh93, Prop. 1.4]. If $\varepsilon_x(u) \neq 0$, then the order of x divides the order of u [MRSW87, Th. 2.7], [Her06, Prop. 3.1]. Moreover the exponent of G and of $\mathrm{V}(\mathbb{Z}G)$ coincide [CL65]. We will use this facts in the following without further mentioning.

Let u be a torsion unit in $V(\mathbb{Z}G)$ of order n and ζ an n-th root of unity in some field K, whose characteristic does not divide n. Let ξ be an (not necessarily primitive) n-th root of unity in K and let φ be a K-representation of G. It was first obtained by Luthar and Passi for K having characteristic 0 [LP89] and later generalized by Hertweck for positive characteristic [Her07] that the multipilicity of ξ as an eigenvalue of $\varphi(u)$, which we denote by $\mu(\xi, u, \varphi)$ and which is of cause a non-negative integer, may be computed as

$$\mu(\xi, u, \varphi) = \frac{1}{n} \sum_{\substack{d|n \\ d\neq 1}} \mathrm{Tr}_{\mathbb{Q}(\zeta)/\mathbb{Q}}(\varphi(u^d)\xi^{-d}) \ + \frac{1}{n} \sum_{\substack{x^G \\ x\ p-\text{regular}}} \varepsilon_x(u) \mathrm{Tr}_{\mathbb{Q}(\zeta)/\mathbb{Q}}(\varphi(x)\xi^{-1}),$$

where as usual $\mathrm{Tr}_{\mathbb{Q}(\zeta)/\mathbb{Q}}(x) = \sum\limits_{\sigma \in \mathrm{Gal}(\mathbb{Q}(\zeta)/\mathbb{Q})} \sigma(x)$.

If u is of prime power order p^k for the first sum in the expression above we obtain

$$\frac{1}{n} \sum_{\substack{d|n \\ d\neq 1}} \mathrm{Tr}_{\mathbb{Q}(\zeta)/\mathbb{Q}}(\varphi(u^d)\xi^{-d}) = \frac{1}{p} \mu(\xi^p, u^p, \varphi).$$

Using these formulas to find possible partial augmentations for torsion units in integral group rings of finite groups is today called HeLP-method. For a diagonalizable matrix A we will write $A \sim (a_1, ..., a_n)$, if the eigenvalues of A, with multiplicities, are $a_1, ..., a_n$.

All subgroups of $G = \mathrm{PSL}(2, p^f)$ were first known to Dickson [Dic01, Theorem 620]. Let $d = \gcd(2, p-1)$. There are cyclic groups of order p, $\frac{p^f+1}{d}$ and $\frac{p^f-1}{d}$ in G and every element of G lies in a conjugate of such a group. The p-Sylow subgroups are elementary-abelian, the Sylow subgroups for all other primes, which are odd, are cyclic and if $p \neq 2$ the 2-Sylow subgroup is dihedral or a Kleinian four-group. There are d conjugacy classes of elements of order p. If $g \in G$ is not of order p or 2 its only distinct conjugate in $\langle g \rangle$ is g^{-1}. Especially there is always only one conjugacy class of involutions. We denote by a a fixed element of order $\frac{p^f-1}{d}$ and by b a fixed element of order $\frac{p^f+1}{d.}$

The modular representation theory of $\mathrm{PSL}(2, q)$ in defining characteristic is well known. All irreducible representations were first given by R. Brauer and C. Nesbitt [BN41]. The explicit Brauer table of $\mathrm{SL}(2, q)$, which contains the Brauer table of $\mathrm{PSL}(2, q)$, may be found in [Sri64]. However, I was not able to find the following Lemma in the literature, except, whitout proof, in Hertwecks preprint [Her07], so a short proof is included.

Lemma 1.2. *Let $G = \mathrm{PSL}(2, p^f)$ and $d = \gcd(2, p-1)$. There are p-modular representations of G given by $\varphi_0, \varphi_1, \varphi_2, ...$ such that there is a $\frac{p^f-1}{d}$-th primitive root of unity α and a $\frac{p^f+1}{d}$-th primitive root of unity β satisfying*

$$\varphi_k(b) \sim (1, \beta, \beta^{-1}, \beta^2, \beta^{-2}, ..., \beta^k, \beta^{-k})$$
$$\varphi_k(a) \sim (1, \alpha, \alpha^{-1}, \alpha^2, \alpha^{-2}, ..., \alpha^k, \alpha^{-k})$$

for every $k \in \mathbb{N}_0$.

Proof: The group $\mathrm{SL}(2, q)$ acts on the vector space spanned by the homogenous polynomials in two commuting variables $x,\ y$ of some fixed degree e extending the natural operation of the 2-dimensional vector space spanned by $x,\ y$, see e.g. [Alp86, p. 14-16]. Since $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} x^i y^j = (-1)^{i+j} x^i y^j$ this action affords a $\mathrm{PSL}(2, q)$-representation if and only if e is even and p is odd or $p = 2$. so let from now on e be even for odd p. Call this representation $\varphi_{\frac{e}{d}}$. Let γ be an eigenvalue of an element in $\mathrm{SL}(2, q)$ mapping onto a under the natural projection from $\mathrm{SL}(2, q)$ to $\mathrm{PSL}(2, q)$. Then $\varphi_{\frac{e}{d}}(a)$ has the same eigenvalues as $\varphi_{\frac{e}{d}}\left(\begin{pmatrix} \gamma & 0 \\ 0 & \gamma^{-1} \end{pmatrix}\right)$. Now $\begin{pmatrix} \gamma & 0 \\ 0 & \gamma^{-1} \end{pmatrix} x^i y^j = \gamma^{i-j} x^i y^j$, so the eigenvalues are $\{\gamma^{i-j} \mid 0 \leq i, j \leq d,\ i + j = e\} = \{(\gamma^d)^t \mid \frac{-e}{d} \leq t \leq \frac{e}{d}\}$. Thus setting $\alpha = \gamma^d$ proves the first part of the claim.
Now let δ be an eigenvalue of an element in $\mathrm{SL}(2, q)$ mapping onto b under the natural projection from $\mathrm{SL}(2, q)$ to $\mathrm{PSL}(2, q)$. The action of $\mathrm{SL}(2, q)$ may of course be extended to $\mathrm{SL}(2, q^2)$. So $\varphi_{\frac{e}{d}}(b)$ has the same eigenvalues as $\varphi_{\frac{e}{d}}\left(\begin{pmatrix} \delta & 0 \\ 0 & \delta^{-1} \end{pmatrix}\right)$, where the matrix may be seen as an element in $\mathrm{SL}(2, q^2)$. Then doing the same calculations as above and setting $\beta = \delta^d$ proves the Lemma.

Using the HeLP-method R. Wagner [Wag95] and Hertweck [Her07] obtained already some results about rational conjugacy of torsion units of prime power order in $\mathrm{PSL}(2, q)$. Part of Wagners result was published in [BHK04].

Lemma 1.3. *[Wag95] Let $G = \mathrm{PSL}(2, p^f)$ and $f \leq 2$. If u is a unit of order p in $V(\mathbb{Z}G)$, then u is rationally conjugate to a group element.*

Remark: The HeLP-method does not suffice to prove rational conjugacy of units of order p in $V(\mathbb{Z}\,\mathrm{PSL}(2, p^f))$ if p is odd and $f \geq 3$. There is also no other method or idea around how one could e.g. obtain, if units of order 3 in $V(\mathbb{Z}\,\mathrm{PSL}(2,27))$ are rationally conjugate to group elements or not.

Lemma 1.4. *[Her07, Prop. 6.4] Let $G = \mathrm{PSL}(2,p^f)$ and let r be a prime different from p. If u is a unit of order r in $V(\mathbb{Z}G)$, then u is rationally conjugate to an element of G.*

Lemma 1.5. *[Her07, Prop. 6.5] Let $G = \mathrm{PSL}(2,p^f)$, let r be a prime different from p and u a torsion unit in $V(\mathbb{Z}G)$ of order r^n. Let $m < n$ and denote by S a set of representatives of conjugacy classes of elements of order r^m in G. Then $\sum\limits_{x\in S} \varepsilon_x(u) = 0$. If moreover g is an element of order r^n in G, then $\mu(1,u,\varphi) = \mu(1,g,\varphi)$ for every p-modular Brauer character φ of G.*

If one is interested not only in cyclic groups the following result is very useful. It may be found e.g. in [Seh93, Lemma 37.6] or in [Val94, Lemma 4].

Lemma 1.6. *Let G be a finite group, U a finite subgroup of $V(\mathbb{Z}G)$ and H a subgroup of G isomorphic to U. If $\sigma : U \to H$ is an isomorphism such that $\chi(u) = \chi(\sigma(u))$ for all $u \in U$ and all irreducible complex characters χ of G, then U is rationally conjugate to H.*

2 Proof of the results

We will first sum up some elementary number theoretical facts. The notatian $a \equiv b \ (c)$ will mean, that a is congruent b modulo c.

Lemma 2.1. *Let t and s be natural numbers such that s divides t and denote by ζ_t and ζ_s a primitive complex t-th root of unity and s-th root of unity respectively. Then*

$$\mathrm{Tr}_{\mathbb{Q}(\zeta_t)/\mathbb{Q}}(\zeta_s) = \mu(s)\frac{\varphi(t)}{\varphi(s)},$$

where μ denotes the Möbius function and φ Euler's totient function. So for a prime r and natural numbers n, m with $m \leq n$ we have

$$\mathrm{Tr}_{\mathbb{Q}(\zeta_{r^n})/\mathbb{Q}}(\zeta_{r^m}) = \begin{cases} r^{n-1}(r-1), & m = 0 \\ -r^{n-1}, & m = 1 \\ 0, & m > 1 \end{cases}$$

Let moreover i and j be integers prime to r, then

$$\mathrm{Tr}_{\mathbb{Q}(\zeta_{r^n})/\mathbb{Q}}(\zeta_{r^m}^i \zeta_{r^m}^{-j}) = \begin{cases} r^{n-1}(r-1), & i \equiv j \ (r^m) \\ -r^{n-1}, & i \not\equiv j \ (r^m), \quad i \equiv j \ (r^{m-1}) \\ 0, & i \not\equiv j \ (r^{m-1}) \end{cases}$$

Proof of Lemma 2.1: Let $s = p_1^{f_1} \cdot ... \cdot p_k^{f_k}$ be the prime factorisation of s. For a natural number l let $I(l) = \{i \in \mathbb{N} \mid 1 \leq i \leq l, \ \gcd(i,l) = 1\}$. As is well known, $\mathrm{Gal}(\mathbb{Q}(\zeta_t)/\mathbb{Q}) = \{\sigma_i : \zeta_t \mapsto \zeta_t^i \mid i \in I(t)\}$. From this the case $s = 1$ follows immediately. Otherwise we have

$$\mathrm{Tr}_{\mathbb{Q}(\zeta_t)/\mathbb{Q}}(\zeta_s) = \sum_{i \in I(t)} \zeta_s^i = \frac{\varphi(t)}{\varphi(s)} \sum_{i \in I(s)} \zeta_s^i = \frac{\varphi(t)}{\varphi(s)} \prod_{j=1}^{k} \sum_{i \in I(p_j^{f_j})} \zeta_{p_j^{f_j}}^i .$$

Now $\sum_{i \in I(p_j^{f_j})} \zeta_{p_j^{f_j}}^i = \begin{cases} -1, & f_j = 1 \\ 0, & f_j > 1 \end{cases}$ and this gives the first formula. The other formulas are special cases of this general formula since $\varphi(r^n) = (r-1)(r^{n-1})$.

Proof of Proposition 1: Let $G = \mathrm{PSL}(2, p^f)$, let r be a prime different from p and let u be a torsion unit in $V(\mathbb{Z}G)$ of order r^n. Let ζ be an r^n-th primitive complex root of unity and set $\mathrm{Tr}_{\mathbb{Q}(\zeta)/\mathbb{Q}} = \mathrm{Tr}$. If $n = 1$, then by Lemma 1.4 u is rationally conjugate to an element in G, so assume $n \geq 2$. Assume further that by induction u^r is rationally conjugate to an element in G. Let m be a natural number such that $m < n$.
We will proceed by induction on m to show that $\varepsilon_x(u) = 0$, if the order of x is r^m. If $m = 0$ this is the Berman-Higman Theorem and if $r = 2$ and $m = 1$ this follows from Lemma 1.5. So assume we know $\varepsilon_x(u) = 0$ for $\circ(x) < r^m$. Let $l = \frac{r^m-1}{2}$ if r is odd and $l = \frac{r^m-2}{2}$ if $r = 2$. Let $\{x_i \mid 1 \leq i \leq l, \ \gcd(i,r) = 1\}$ be a full set of representatives of conjugacy classes of elements of order r^m in G such that $x_1^i = x_i$ (this is possible by the group theoretical properties of G given above).
We will prove by induction on k that $\varepsilon_{x_i}(u) = \varepsilon_{x_j}(u)$ for $i \equiv \pm j \ (r^{m-k})$. This is certainly true for $k = 0$ and once we establish it for $k = m$, if r is odd, and $k = m-1$, if $r = 2$, it will follow from Lemma 1.5 that $\varepsilon_{x_i}(u) = 0$ for all i. So assume $\varepsilon_{x_i}(u) = \varepsilon_{x_j}(u)$ for $i \equiv \pm j \ (r^{m-k})$. Since u^r is rationally conjugate to a group element, there exists a primitive r^{n-1}-th root of unity $\zeta_{r^{n-1}}$ such that

$$\varphi_{r^k}(u^r) \sim (1, \zeta_{r^{n-1}}, \zeta_{r^{n-1}}^{-1}, \zeta_{r^{n-1}}^{2}, \zeta_{r^{n-1}}^{-2}, ..., \zeta_{r^{n-1}}^{r^k}, \zeta_{r^{n-1}}^{-r^k}).$$

Now all p-modular Bruaer characters of G are real valued and thus we obtain that $\varphi_{r^k}(u) \sim (1, a_1, a_1^{-1}, a_2, a_2^{-1}, ..., a_{r^k}, a_{r^k}^{-1})$, where for every i we have a_i a root of unity such that $a_i^{r^{m-k}} \neq 1$. So for every primitive r^{m-k}-th root of unity $\zeta_{r^{m-k}}$ we have $\mu(\zeta_{r^{m-k}}, u, \varphi_{r^k}) = 0$. Let ζ_{r^m} be a primitive r^m-th root of unity such that we have $\varphi_{r^k}(x_1) \sim (1, \zeta_{r^m}, \zeta_{r^m}^{-1}, ..., \zeta_{r^m}^{r^k}, \zeta_{r^m}^{-r^k})$ and set $\xi = \zeta_{r^m}^{r^k}$. Let S be a set of representatives of elements of G of r-power order not greater than r^n containing $x_1, ..., x_l$ and let moreover α be a natural number prime to r such that $1 \leq \alpha \leq l$.
Thus $\mu(\xi^\alpha, u, \varphi_{r^k}) = 0$ and $\varepsilon_x(u) = 0$ for $\circ(x) < r^m$. From here on a sum over i will always mean a sum over all defined i, that will be $1 \leq i \leq l$ and $r \nmid i$. Then using the HeLP-method we get

$$\begin{aligned}
0 = \mu(\xi^\alpha, u, \varphi_{r^k}) &= \frac{1}{r}\mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) + \frac{1}{r^n}\sum_{x \in S} \varepsilon_x(u)\mathrm{Tr}(\varphi_{r^k}(x)\xi^{-\alpha}) \\
&= \frac{1}{r}\mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) + \frac{1}{r^n}\sum_{\substack{x \in S \\ \circ(x) > r^m}} \varepsilon_x(u)\mathrm{Tr}(\varphi_{r^k}(x)\xi^{-\alpha}) + \frac{1}{r^n}\sum_i \varepsilon_{x_i}(u)\mathrm{Tr}(\varphi_{r^k}(x_i)\xi^{-\alpha}) \\
&= \frac{1}{r}\mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) + \frac{1}{r^n}\sum_{x \in S} \varepsilon_x(u)\mathrm{Tr}(\xi^{-\alpha}) + \frac{1}{r^n}\sum_i \varepsilon_{x_i}(u)\mathrm{Tr}((\xi^i + \xi^{-i})\xi^{-\alpha}) \\
&= \frac{1}{r}\mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) + \frac{\mathrm{Tr}(\xi^{-\alpha})}{r^n} + \frac{1}{r^n}\sum_i \varepsilon_{x_i}(u)\mathrm{Tr}((\xi^i + \xi^{-i})\xi^{-\alpha}). \qquad (1)
\end{aligned}$$

In the third line we used that if $\tilde{\zeta}$ is a root of unity of r-power order such that $\tilde{\zeta}^{r^{m-k}} \neq 1$, then $\tilde{\zeta}\xi$ has the same order as $\tilde{\zeta}$ and so $\mathrm{Tr}(\tilde{\zeta}\xi) = 0$ by Lemma 2.1. Note that as i is prime to r the congruence $i \equiv \alpha \ (r^{m-k})$ implies $-i \not\equiv \alpha \ (r^{m-k})$ for $r^{m-k} \notin \{1, 2\}$ and these exceptions don't have to be considered by our assumptions on m and k.
There are now two cases to consider. First assume $k < m - 1$, so ξ is at least of order r^2. Then we have $\mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) = 0$ and using Lemma 2.1 in (1) we obtain

$$\begin{aligned}
0 &= \frac{1}{r^n}\sum_i \varepsilon_{x_i}(u)\mathrm{Tr}((\xi^i + \xi^{-i})\xi^{-\alpha}) \\
&= \frac{1}{r^n}\sum_{i \equiv \pm\alpha (r^{m-k})} \varepsilon_{x_i}(u)(r^{n-1}(r-1)) + \frac{1}{r^n}\sum_{\substack{i \equiv \pm\alpha (r^{m-k-1}) \\ i \not\equiv \pm\alpha (r^{m-k})}} \varepsilon_{x_i}(u)(-r^{n-1}) \\
&= \sum_{i \equiv \pm\alpha (r^{m-k})} \varepsilon_{x_i}(u) - \frac{1}{r}\sum_{i \equiv \pm\alpha (r^{m-k-1})} \varepsilon_{x_i}(u). \qquad (2)
\end{aligned}$$

So

$$r\sum_{i\equiv\pm\alpha(r^{m-k})}\varepsilon_{x_i}(u)=\sum_{i\equiv\pm\alpha(r^{m-k-1})}\varepsilon_{x_i}(u).$$

But since by induction $\varepsilon_{x_i}(u)=\varepsilon_{x_j}(u)$ for $i\equiv\pm j\ (r^{m-k})$ the summands on the left hand side are all equal and since changing α by r^{m-k-1} does not change the right hand side of the equation we get $\varepsilon_{x_i}(u)=\varepsilon_{x_j}(u)$ for $i\equiv\pm j\ \ (r^{m-k-1})$.
Now consider $k=m-1$, then ξ is a primitive r-th root of unity and thus we have $\mu(\xi^{\alpha r},u^r,\varphi_{r^k})=1$. So using Lemma 2.1 in (1) we get

$$\begin{aligned}0&=\frac{1}{r}+\frac{-r^{n-1}}{r^n}+\frac{1}{r^n}\sum_{\pm i\not\equiv\alpha(r)}\varepsilon_{x_i}(u)(-2r^{n-1})+\frac{1}{r^n}\sum_{\pm i\equiv\alpha(r)}\varepsilon_{x_\alpha}(u)(r^{n-1}(r-1)-r^{n-1})\\&=\sum_{\pm i\equiv\alpha(r)}\varepsilon_{x_i}(u)-\frac{2}{r}\sum_{i}\varepsilon_{x_i}(u).\end{aligned}\tag{3}$$

So

$$r\sum_{\pm i\equiv\alpha(r)}\varepsilon_{x_i}(u)=2\sum_{i}\varepsilon_{x_i}(u).$$

Now by Lemma 1.5 the right side of this equation is zero and by induction all summands on the left side are equal. Hence varying α gives $\varepsilon_x(u)=0$ for $\circ(x)=r^m$.

So it only remains to show that $\varepsilon_x(u)=1$ for exactly one conjugacy class x^G in G, where $\circ(x)=r^n$. The arguments in this case are very close to the arguments above. Let $k\leq n$. As in the computation above we have $\varphi_{r^k}(u^r)\sim(1,\zeta_{r^{n-1}},\zeta_{r^{n-1}}^{-1},...,\zeta_{r^{n-1}}^{r^k},\zeta_{r^{n-1}}^{-r^k})$ for some primitive r^{n-1}-th root of unity and $\varphi_{r^k}(u)\sim(1,a_1,a_1^{-1},a_2,a_2^{-1},...,a_{r^k},a_{r^k}^{-1})$, where a_i are roots of unity such that $a_i^{r^{n-k}}\neq 0$ for $1\leq i\leq r^k-1$ and a_{r^k} is some primitive r^{n-k}-th root of unity. Set $\xi=a_{r^k}$ and let $l=\frac{r^n-1}{2}$, if r is odd, and $l=\frac{r^n-2}{2}$, if $r=2$. Let $\{x_i\mid 1\leq i\leq l,\ \gcd(i,r)=1\}$ be a set a representatives of conjugacy classes of elements of order r^n in G such that $x_i=x_1^i$ and $\varphi_1(x_1)\sim\varphi_1(u)$. Then x_1^r is rationally conjugate to u^r. We will prove by induction on k that:

(i) $\varepsilon_{x_1}(u)=1$ and $\varepsilon_{x_i}(u)=0$ for $i\equiv\pm1\ \ (r^{n-k}), i\neq 1$.

(ii) $\varepsilon_{x_i}(u)=\varepsilon_{x_j}(u)$ for $i\equiv\pm j\ \ (r^{n-k})$ and $i\not\equiv\pm1\ \ (r^{n-k})$.

We will prove these two facts for $k=n-1$. If $r=2$, then the Proposition will follow from this. If r is odd, we will prove afterwards that $\sum\limits_{i\equiv\alpha(r)}\varepsilon_{x_i}(u)=0$ for $\alpha\not\equiv\pm1\ (r)$, which then also implies the Proposition.

Let α be a natural number prime to r with $1 \leq \alpha \leq l$. Using the HeLP-method and $\varepsilon_x(u) = 0$ for $\circ(x) < r^n$ we obtain, doing the same calculations as in (1):

$$\mu(\xi^\alpha, u, \varphi_{r^k}) = \frac{1}{r}\mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) + \frac{\mathrm{Tr}(\xi^{-\alpha})}{r^n} + \frac{1}{r^n}\sum_i \varepsilon_{x_i}(u)\mathrm{Tr}((\xi^i + \xi^{-i})\xi^{-\alpha}). \qquad (4)$$

As u^r is rationally conjugate to x_1^r we know that $\xi^{\pm r}$ are eigenvalues of $\varphi_{r^k}(u^r)$. So we get

$$\mu(\xi^\alpha, u, \varphi_{r^k}) = \begin{cases} 1, & \alpha \equiv \pm 1 \ (r^{n-k}) \\ 0, & \text{else} \end{cases} \quad \text{and} \quad \mu(\xi^{\alpha r}, u^r, \varphi_{r^k}) = \begin{cases} 1, & \alpha \equiv \pm 1 \ (r^{n-k-1}) \\ 0, & \text{else} \end{cases}$$

There are now several cases to consider: (ii) is clear for $k = 0$ and if $\alpha \not\equiv \pm 1 \ (r^{n-k})$ we can do the same computations as in (2) to obtain (ii), if $k < n-1$. So (ii) holds for $k = n-1$.

To obtain the base case for (i) set $k = 0$. Then from (4) we obtain (similar to the computation in (2)):

$$1 = \frac{1}{r} + \varepsilon_{x_1}(u) - \frac{1}{r}\sum_{i \equiv \pm 1 (r^{n-1})} \varepsilon_{x_i}(u)$$

and

$$0 = \frac{1}{r} + \varepsilon_{x_\alpha}(u) - \frac{1}{r}\sum_{i \equiv \pm 1 (r^{n-1})} \varepsilon_{x_i}(u)$$

for $\alpha \equiv \pm 1 \ (r^{n-1})$ and $\alpha \neq 1$. Substracting two such equations gives

$$1 = \varepsilon_{x_1}(u) - \varepsilon_{x_\alpha}(u) \qquad (5)$$

for every $\alpha \equiv \pm 1 \ (r^{n-1})$ and $\alpha \neq 1$. Let $t = |\{i \in \mathbb{N} | i \leq l, i \equiv \pm 1 \ (r^{n-1})\}|$. Then summing up the equations for all $\alpha \equiv \pm 1 \ (r^{n-1})$ gives

$$1 = \frac{t}{r} + \sum_{i \equiv \pm 1 (r^{n-1})} \varepsilon_{x_i}(u) - \frac{t}{r}\sum_{i \equiv \pm 1 (r^{n-1})} \varepsilon_{x_i}(u) = \frac{t}{r} + (1 - \frac{t}{r})\sum_{i \equiv \pm 1 (r^{n-1})} \varepsilon_{x_i}(u).$$

So $\sum_{i \equiv \pm 1 (r^{n-1})} \varepsilon_{x_i}(u) = 1$ and the base case of (i) follows from (5).

So assume $1 \leq k < n-1$. Then $\sum_{i \equiv \pm 1 \ (r^{n-k})} \varepsilon_{x_i}(u) = 1$ by induction and for $\alpha \equiv \pm 1 \ (r^{n-k})$

from (4) computing as in (2) we obtain

$$1 = \frac{1}{r} + \sum_{i \equiv \pm 1 (r^{n-k})} \varepsilon_{x_i}(u) - \frac{1}{r} \sum_{i \equiv \pm 1 (r^{n-k-1})} \varepsilon_{x_i}(u) = \frac{1}{r} + 1 - \frac{1}{r} \sum_{i \equiv \pm 1 (r^{n-k-1})} \varepsilon_{x_i}(u).$$

For $\alpha \not\equiv \pm 1 \ (r^{n-k})$ and $\alpha \equiv \pm 1 \ (r^{n-k-1})$ we obtain the same way

$$0 = \frac{1}{r} + \sum_{i \equiv \pm \alpha (r^{n-k})} \varepsilon_{x_i}(u) - \frac{1}{r} \sum_{i \equiv \pm 1 (r^{n-k-1})} \varepsilon_{x_i}(u).$$

Thus subtracting the last equation from the one before gives

$$1 = 1 - \sum_{i \equiv \pm \alpha (r^{n-k})} \varepsilon_{x_i}(u).$$

The summands on the right hand side are all equal by (ii), so $\varepsilon_{x_\alpha}(u) = 0$, as claimed. Finally let r be odd, $k = n-1$ and $\alpha \not\equiv \pm 1 \ (r)$. Then $\mu(\xi^\alpha, u^r, \varphi_{r^k}) = \mu(1, u^r, \varphi_{r^k}) = 3$. So from (4) computing as in (3) we obtain

$$0 = \frac{3}{r} + \frac{-r^{n-1}}{r^n} - \frac{2}{r} \sum_i \varepsilon_{x_i}(u) + \sum_{i \equiv \pm \alpha (r)} \varepsilon_{x_i}(u) = \sum_{i \equiv \pm \alpha (r)} \varepsilon_{x_i}(u).$$

As by (ii) all summands in the last sum are equal, we get $\varepsilon_{x_\alpha}(u) = 0$ and the Proposition is finally proved.

Proof of Theorem 2: Let $G = \mathrm{PSL}(2, p^f)$ such that $f \leq 2$ or $p = 2$. Assume first that r is an odd prime, which is not p, and R is an r-subgroup of $V(\mathbb{Z}G)$. As every r-subgroup of G is cyclic so is R by [Her08b, Theorem A] and thus R is rationally conjugate to a subgroup of G by Proposition 1. If $p \neq 2$ and R is a 2-subgroup of $V(\mathbb{Z}G)$, then R is either cyclic or dihedral or a Kleinian four group by [HHK09, Theorem 2.1]. If R is cyclic, then it is rationally conjugate to a subgroup of G by Proposition 1. If R is dihedral or a Kleinian four group let $S = \langle s \rangle$ be a maximal cyclic subgroup of R. Then s is rationally conjugate to an element $g \in G$ by Proposition 1. Moreover R is isomorphic to some subgroup of H of G, such that the maximal cyclic subgroup of H is generated by g. As there is only one conjugacy class of involutions in G every isomorphism σ between R and H mapping s to g satisfies $\chi(\sigma(u)) = \chi(u)$ for every irreducible complex character of G. Thus R is rationally conjugate to H by Lemma 1.6.

If $p = 2$ and P is a 2-subgroup of $V(\mathbb{Z}G)$ then all non-trivial elements of P are involutions, so P is elementary abelian. As there is again only one conjugacy class of involutions in G every isomorphism σ between P and a subgroup of G isomorphic with P satisfies $\chi(\sigma(u)) = \chi(u)$ for every irreducible complex character of G. So P is rationally conjugate to a subgroup of G by Lemma 1.6. Finally assume that p is odd and P is a p-subgroup of $V(\mathbb{Z}G)$. If P is of order p it is rationally conjugate to a subgroup of G by Lemma 1.3. If P is of order p^2, it is elementary abelian. Let c and d be generators of P, then they are rationally conjugate to group elements by Lemma 1.3. But there are only two conjugacy classes of elements of order p and to whichever elements c and d are conjugate, it is possible to pick some, which generate an elementary abelian subgroup of G of order p^2. Then again we obtain an isomorphism σ preserving character values.

Remark: Let $G = \mathrm{PSL}(2, p^f)$ and let n be a number prime to p. The structure of the Brauer table of G in defining characteristic yields immidiately, that if we can prove that a unit $u \in V(\mathbb{Z}G)$ of order n is rationally conjugate to an element in G applying the HeLP-method to the Brauer table, then this calculations will hold over any $\mathrm{PSL}(2, q)$, if n and q are coprime. In this sense it would be interesting, and seems actually achievable, to determine a subset A_{p^f} of $\mathbb{N}$ such that we can say: The HeLP-method proves that a unit $u \in V(\mathbb{Z}G)$ of order n is rationally conjugate to an element in G if and only if $n \in A_{p^f}$. Test computations yield the conjecture that A_{p^f} actually contains all odd numbers prime to p. If this turned out to be true this would yield, using the results in [Her07], the First Zassenhaus Conjecture for the groups $\mathrm{PSL}(2, p)$, where p is a Fermat- or Mersenne prime.
Other interesting questions concerning torsion units of the integral group ring of $G = \mathrm{PSL}(2, p^f)$ were mentioned at the end of [HHK09] and are still open today: If the order of $u \in V(\mathbb{Z}G)$ is divisable by p, is u of order p? Are units of order p rationally conjugate to elements of G? Are there non-abelian p-subgroups in $V(\mathbb{Z}G)$?

Acknowledgement: The computations given above were all done by hand, but some motivating computations were done using a GAP-implementation of the HeLP-algorithm written by Andreas Bächle.

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