Long time decay estimates for dissipative evolution equations

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Abstract.
In the first part of the talk I will consider the following Cauchy problem for a wave equation with time-dependent damping term $b(t)u_t$ and mass term $m(t)^2u$, and a power nonlinearity $|u|^p$:

$$\begin{cases} u_{tt} - \Delta u + b(t)u_t + m^2(t)u = |u|^p, \quad t \geq 0, \ x \in \mathbb{R}^n, \\ u(0,x) = f(x), \quad u_t(0,x) = g(x). \end{cases} \tag{1}$$

I will discuss how the interplay between an effective time-dependent damping term and a time-dependent mass term influences the decay rate of the solution to the corresponding linear Cauchy problem. I will consider the case in which the mass is dominated by the damping term, i.e. $m(t) = o(b(t))$ as $t \to \infty$, and the case in which the mass is dominant i.e. $\liminf_{t\to\infty} (m(t)/b(t)) > 1/4$.
Then I will show how to use the estimates of solutions to the linear Cauchy problem to prove the existence of global in-time energy solutions to the non-linear Cauchy problem (1), in a supercritical range $p > \bar{p}$, assuming small data in the energy space $(f, g) \in H^1 \times L^2$, possibly with additional regularity L^η for the data.
In the second part of the talk, I will show $L^1 - L^1$ long time estimates for the strongly damped plate equation

$$u_{tt} + \Delta^2 u + \Delta^2 u_t = 0 \quad x \in \mathbb{R}^n, \, t \in \mathbb{R}_+, \quad u(0,x) = u_0(x), \ u_t(0,x) = u_1(x).$$

References

[1] M. D'Abbicco, G. Girardi, M. Reissig: A scale of critical exponents for semilinear waves with time-dependent damping and mass terms, Nonlinear Analysis, **179**, 15-40 (2019), https://doi.org/10.1016/j.na.2018.08.006.

[2] M. D'Abbicco, G. Girardi, J. Liang: $L^1 - L^1$ estimates for the strongly damped plate equation, submitted.