Spectral Theory and Dynamics of Quantum Systems

GRADUIERTENKOLLEG 1838

Guest Lecture

Prof. Dirk Hundertmark

KIT Karlsruhe

The solution of the Gevrey smoothing conjecture for the non-cutoff homogenous Boltzmann equation for Maxwellian molecules

November 5, 2018 - 17:00 h
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University of Stuttgart, Pfaffenwaldring 57

Abstract: It has long been suspected that the non-cutoff Boltzmann operator has similar coercivity properties as a fractional Laplacian. This has led to the hope that the homogenous Boltzmann equation enjoys similar regularity properties as the heat equation with a fractional Laplacian. In particular, the weak solution of the non-cutoff homogenous Boltzmann equation with initial datum in $L^1_2(\mathbb{R}^3) \cap L\log L(\mathbb{R}^3)$ i.e., finite mass, energy and entropy, should immediately become Gevrey regular. So far, the best available results show that the solution becomes H^∞ regular for positive times. Gevrey regularity is also known for weak solutions of the linearised Boltzmann equation, where one studies solutions close to a Maxwellian distribution, or under additional decay assumptions on the solutions. The main problem for establishing Gevrey regularity is that, in order to use the coercivity results on the non-cutoff Boltzmann collision kernel, one has to bound a non-linear and non-local commutator of the Boltzmann kernel with certain sub-exponential weights. We will show why the previous H^∞ smoothing results for the non-cutoff Boltzmann equation are so much simpler than proving the (optimal) Gevrey regularity. In fact, we will try to convince the audience that a proof of Gevrey regularity should be impossible, which is of course wrong, given that we have a proof of Gevrey regularization under minimal physically reasonable assumptions. This is joint work with Jean-Marie Barbaroux, Tobias Ried, and Semjon Wugalter.

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