

$$\inf_{W \in W(\mathcal{G})} \sup_{V \in V(\mathcal{G})} \frac{B[W, V]}{\|W\|_W \|V\|_V} > 0$$

Colloquium in Memory of Prof. Anna-Margarete Sändig

March 27, 2025

Seminarraum 8.122

$$f(u_+) - f(u_-)$$

```
39 typedef Dune::ACFem::MassModel<EllipticModelType> MassModelType;
40 MassModelType bareMassModel(implicitEllipticModel);
41
42 auto massModel(mu * (mat.Z_a) * J + mat.Z_w) * bareMassModel();
```

having a coffee in a Banach space makes you feel complete

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|-------------|---|
| 9:00–9:15 | Prof. Dominik Göldeke, Prof. Christian Rohde (University of Stuttgart)
Opening |
| 9:15–10:00 | Prof. Wolfgang Wendland (University of Stuttgart)
Anna Sändig in Stuttgart |
| 10:00–10:45 | Prof. Thomas Apel (UniBw München)
Weak and very weak formulations of the Stokes problem with non-homogeneous boundary condition |
| 10:45–11:15 | Coffee break |
| 11:15–12:00 | Prof. Dorothee Knees (University of Kassel)
Regularity results for a static relaxed micromorphic model |
| 12:00–14:00 | Lunch |
| 14:00–14:45 | Prof. Jürgen Roßmann (University of Rostock)
A mixed problem for the nonstationary Stokes system |
| 14:45–15:30 | Ph.D. Monika Balázsová (Institute of Math., Czech Academy of Sciences)
Semilinear parabolic initial boundary value problems with nonlinear Newton boundary conditions |
| 15:30–16:15 | Prof. Marita Thomas (Free University and Weierstrass Institute, Berlin)
Analysis of a dynamic phase-field fracture model and how spaghettis break |
| 16:15–18:00 | Get together |

Alle Interessenten sind herzlich eingeladen!

Institut für Angewandte Analysis und Numerische Simulation

Veranstaltungsort: Raum 8.122, Pfaffenwaldring 57
70569 Stuttgart

<http://www.ians.uni-stuttgart.de>

$$\|U - u\|_W \lesssim \sum_{E \in \mathcal{E}_h} \left(\varepsilon_E \|\nabla \cdot f(u)\| + \partial_t u + \operatorname{div}_x f(u) = 0 \right)$$

Weak and very weak formulations of the Stokes problem with non-homogeneous boundary condition

Thomas Apel (UniBw München)

The presentation focuses on the homogeneous Stokes equations with non-homogeneous boundary conditions, and discusses how to understand the solution when the Dirichlet data are non-smooth such as in $L^2(\Gamma)$. A weak solution $(u, p) \in H^1(\Omega)^2 \times L^2(\Omega)$ cannot be expected to exist. Instead, a very weak formulation is defined, wherein the solution is sought in $L^2(\Omega)^2 \times H^{-1}(\Omega)$. The corner singularities are studied in order to obtain slightly more regularity than $L^2(\Omega)^2 \times H^{-1}(\Omega)$. Previous results on that topic are restricted to convex domains where the dual problem has a solution in $H^2(\Omega)^2 \times H^1(\Omega)$ which is not valid when non-convex domains are considered.

Regularity results for a static relaxed micromorphic model

Dorothee Knees (University of Kassel)

The relaxed micromorphic model is a generalised continuum model allowing to describe for instance size effects of microstructured solids. The state of the solid subject to external loads is characterized by the displacement field $u : \mathbb{R}^3 \supset \Omega \rightarrow \mathbb{R}^3$ and the microdistortion tensor $P : \Omega \rightarrow \mathbb{R}^{3 \times 3}$. The corresponding system of partial differential equations consist of the system of linear elasticity that is coupled with a system of Maxwell type for the distortion tensor P . We will discuss the regularity of weak solutions of linear and nonlinear versions of this model under different assumptions on the smoothness of the domain. The main ingredients for the proofs are the Helmholtz decomposition and refined difference quotient arguments based on generalized inner variations in combination with a Piola-type transformation. This is joint work with Patrizio Neff (Duisburg-Essen) and Sebastian Owczarek (Warsaw).

A mixed problem for the nonstationary Stokes system

Jürgen Roßmann (University of Rostock)

Elliptic problems in domains with angular or conical points on the boundary are studied for a long time. Several authors were interested particularly in boundary value problems for the Stokes and Navier-Stokes systems. Anna Sändig was one of them. In a paper with M. Orlt she studied the solutions of a mixed boundary value problem in a polygon, where very general boundary conditions were given on the different parts of the boundary.

In this talk, we consider a mixed initial-boundary value problem for the nonstationary Stokes system in a polygon Ω , where Dirichlet and Neumann conditions are prescribed on the different sides of Ω . We start with the parameter-dependent problem which arises after the use of the Laplace transform and obtain estimates for the solutions in the class of the weighted Sobolev spaces $V_\beta^l(\Omega)$ which are defined as the set of all functions (vector functions) such that $\prod r_j^{\beta_j - l + |\alpha|} \partial_x^\alpha u \in L_2(\Omega)$ for $|\alpha| \leq l$. Here r_j is the distance from the corner point P_j and β_j is the j th component of β . These estimates enable one to prove the existence of solutions (u, p) of the nonstationary problem, where $u \in L_2(0, T; V_\beta^2(\Omega))$, $u_t \in L_2(0, T; V_\beta^0(\Omega))$ and $\nabla p \in L_2(0, T; V_\beta^0(\Omega))$.

Semilinear parabolic initial boundary value problems with nonlinear Newton boundary conditions

Monika Balázsová (Institute of Mathematics, Czech Academy of Sciences)

In this talk we will consider a parabolic evolution equation with a nonlinear Newton boundary condition in a polygonal space-time cylinder. First we show, that the elliptic part is given by a m -accretive mapping, therefore we can apply the theory of nonlinear semigroups in Banach spaces in order to get regularity results in time and space. In the second part of the talk we will solve this problem numerically using the space-time discontinuous Galerkin method and we derive an abstract error estimate.

Analysis of a dynamic phase-field fracture model and how spaghetti's break

Marita Thomas (Free University and Weierstrass Institute, Berlin)

Why does a spaghetti usually break into more than just two pieces? We investigate this observation by means of a phase-field model for dynamic crack propagation. Herein, the sharp crack surface is regularized by means of a phase-field approximation and a non-smooth penalization prevents material healing. The momentum balance is formulated as a first order system and coupled in a nonlinear way to the non-smooth phase-field evolution. We address this modeling approach both in the small and in the finite strain setting and introduce suitable notions of weak solutions. We discuss the existence of weak solutions by means of a fully discrete approximation of the problem in space and time and study their behavior in numerical simulations.

This is joint work with Sven Tornquist (WIAS), Christian Wieners (Karlsruhe), and Kerstin Weinberg (Siegen), financially supported by the German Research Foundation (DFG) in the priority programme "Variational Methods for Predicting Complex Phenomena in Engineering Structures and Materials (SPP 2256)" within the project "Nonlinear Fracture Dynamics: Modeling, Analysis, Approximation, and Applications".