The recent experimental advances in generating intense and short-lived THz light pulses opened the way for a plethora of potential ground-breaking applications. Indeed, the THz spectral window corresponds to the timescale of several collective excitations in solids, ranging from lattice vibrations to collective modes of the electron system across a phase transitions, like e.g. the superconducting one. From the theoretical point of view, facing with time-resolved spectroscopy with intense THz light pulses requires to address simultaneously two complementary problems: going beyond the linear-response regime, valid for weak perturbing fields, and describing a purely out-of-equilibrium process, where one probes relaxation out of a (possible) metastable state. While addressing the latter problem is still at its infancy, in the last decade several progresses have been done on the former aspect. In this talk I will review our contribution to the understanding of resonant and non-resonant excitations driven by strong THz fields in solids. Starting from the well-understood case of phonon modes in insulators, I will address the peculiar behavior of collective electronic excitations in superconductors. I will show how a common theoretical framework can be efficiently used to interpret several experimental findings, making non-linear THz spectroscopy an extremely versatile tool to disentangle the microscopic processes at play in different systems.